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CONFIDENTIAL

SET A

UNIVERSITI KUALA LUMPUR
Malaysia France Institute

FINAL EXAMINATION
JANUARY 2010 SESSION

SUBJECT CODE	:	FKB 14202
SUBJECT TITLE	:	ENGINEERING MATHEMATICS 2
LEVEL	:	BACHELOR
TIME / DURATION	:	8.00pm – 10.00pm (2 HOURS)
DATE	:	05 MAY 2010

INSTRUCTIONS TO CANDIDATES

1. Please read the instructions given in the question paper CAREFULLY.
2. This question paper is printed on both sides of the paper.
3. Please write your answers on the answer booklet provided.
4. Answer should be written in blue or black ink except for sketching, graphic and illustration.
5. This questions paper consists of SIX (6) questions. Answer FOUR (4) questions only.
6. Answer ALL questions in English.

THERE ARE 5 PAGES OF QUESTIONS AND 2 PAGES OF FORMULA, EXCLUDING THIS PAGE.

(Total: 60 marks)

INSTRUCTION: Answer only FOUR questions.
Please use the answer booklet provided.

Question 1

(a) A town is in the shape of a rectangle that is 2 miles by 1 miles. At its center is an industrial plant that emits pollutants into the air. The concentration C of these pollutants is modeled by the formula : $C = 120 - 4x^2 - 4y^2$ where the distance of a point (x,y) from the town's center is measured in miles. If the average concentration of pollutants in the air throughout the town is given by $\frac{1}{2}\int_{-1/2}^{1/2}\int_{-1}^{1} C\,dxdy$. Calculate the average concentration of pollutants in the air.

(7 marks)

(b) The volume of the frustum of a cone (refer Figure 1) is given by the function :

$$V(R, r, h) = \frac{1}{3}\pi h\left(R^2 + Rr + r^2\right).$$

Calculate the **approximate change** in the volume of the frustum of a cone if the upper radius r is decreased from 3 to 2.7 centimeters, the base radius R is increased from 8 cm to 8.1 cm, and the height h is increased from 6 cm to 6.3 cm.

(8 marks)

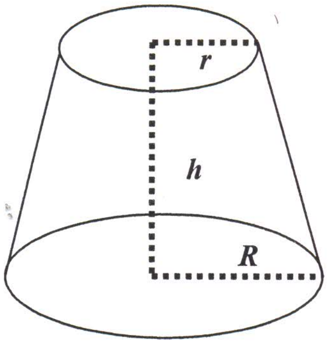

Figure 1

Question 2

Evaluate the double integral $\iint_R f(x,y)\,dA$ for the function $f(x,y) = 4e^{x^2} - 5\sin y$ where R is the region bounded by the graphs $y = x,\quad y = 0,\quad x = 4$.

(15 marks)

Question 3

A packaging company must design an open cardboard box with a total volume of $64\,ft^3$. The box will have a bottom, no top, and two parallel partitions that divide the box into three equal sections (refer Figure 2).

(a) Show that the amount of cardboard used is given by :

$$S(x,y) = xy + \frac{256}{y} + \frac{128}{x}$$

(3 marks)

(b) Find the dimensions of the box that will use the least amount of cardboard.

(12 marks)

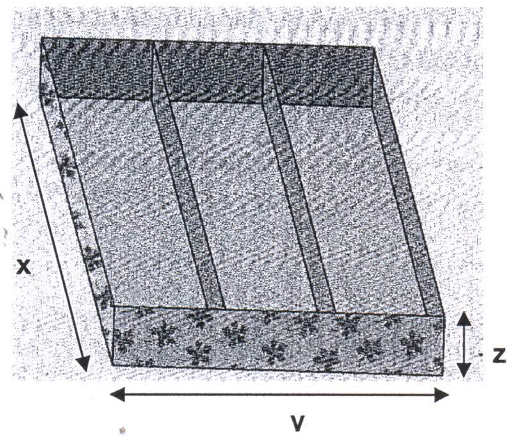

Figure 2

Question 4

Consider a block of mass 50 kg sliding down an incline of slope 30°, as illustrated in figure 3. The block starts from the rest on the slope at time $t = 0$. If the coefficient of friction is 0.4 and the force opposing motion due to air resistance is $0.5v$, where v is the velocity. Newton's second law gives $50\dfrac{dv}{dt} = 75 - 0.5v$. Solve the given differential equation to derive an equation stating how the velocity v, down the slope will vary with time, t if $v = 0$ when $t = 0$.

(15 marks)

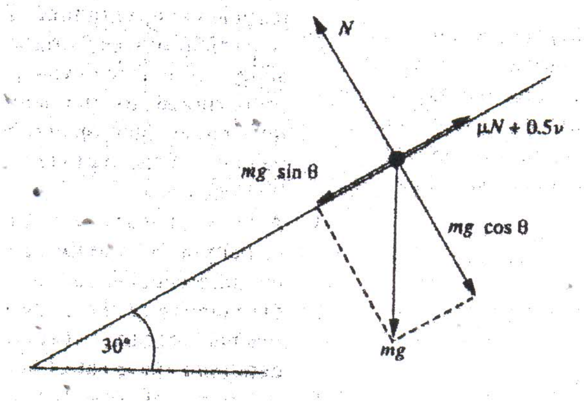

Figure 3

Question 5

Given the particular integral of differential equation :

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 4x + 3\cos 2x$$

is in the form of $f(x) \equiv ax + b + c\cos 2x + d\sin 2x$.

a) Find the value of *a, b, c* and *d* of the particular integral.

(11 marks)

b) Hence, determine the general solution of the given differential equation :

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 4x + 3\cos 2x$$

(4 marks)

Question 6

(a) Consider a mechanical system where a vertically suspended spring with a mass m attached to its lower end, as illustrated by figure 4. The mass is pulled down some distance x and then released. Given the mechanical system is corresponding to the differential equation : $m\frac{d^2x}{dt^2} = -kx$ where k and m are constants. Determine general solution of the given differential equation.

(5 marks)

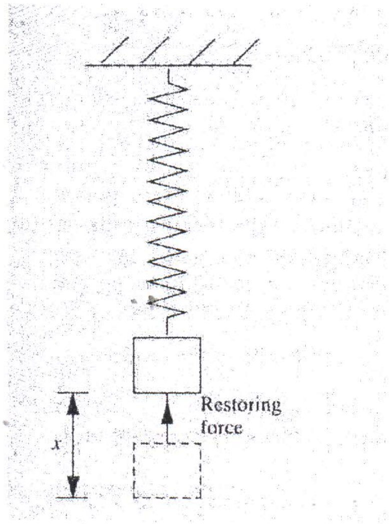

Figure 4

(b) $L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = 0$ is an equation representing current i in an electric circuit. If inductance L is 0.25 henry, capacitance C is 29.76×10^{-6} farads and R is 250 ohms, solve the equation for i given the boundary conditions that when $t = 0,\quad i = 0 \text{ and } \frac{di}{dt} = 34.$

(10 marks)

END OF QUESTION

DIFFERENTIATION

TRIGONOMETRIC FUNCTIONS

STANDARD FORM	GENERALFORM
$\frac{d}{dx}(\sin x) = \cos x$	$\frac{d}{dx}(\sin f(x)) = \cos f(x) f'(x)$
$\frac{d}{dx}(\cos x) = -\sin x$	$\frac{d}{dx}(\cos f(x)) = -\sin f(x) f'(x)$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\frac{d}{dx}(\tan f(x)) = \sec^2 f(x). f'(x)$
$\frac{d}{dx}(\csc x) = -\csc x \cot x$	$\frac{d}{dx}(\csc f(x)) = -\csc f(x) \cot f(x). f'(x)$
$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\frac{d}{dx}(\sec f(x)) = \sec f(x) \tan f(x). f'(x)$
$\frac{d}{dx}(\cot x) = -\csc^2 x$	$\frac{d}{dx}(\cot f(x)) = -\csc^2 f(x). f'(x)$

EXPONENTIAL FUNCTION

STANDARD FORM	GENERALFORM
$\frac{d}{dx} e^x = e^x$	$\frac{d}{dx} e^{f(x)} = e^{f(x)}. f'(x)$

LOGARITHMIC FUNCTION

STANDARD FORM	GENERALFORM
$\frac{d}{dx} \ln x = \frac{1}{x}$	$\frac{d}{dx} \ln f(x) = \frac{f'(x)}{f(x)}$

INTEGRATION - BASIC RULES

(trigonometric functions , exponential functions , form 1/x)

STANDARD FORM trigonometric	GENERAL FORM Where : $f(x)=ax+b$
$\int \cos x dx = \sin x + c$	$\int \cos f(x) dx = \frac{\sin f(x)}{f'(x)} + c$
$\int \sin x dx = -\cos x + c$	$\int \sin f(x) dx = \frac{-\cos f(x)}{f'(x)} + c$
$\int \sec^2 x dx = \tan x + c$	$\int \sec^2 f(x) dx = \frac{\tan f(x)}{f'(x)} + c$
$\int \sec x \tan x dx = \sec x + c$	$\int \sec f(x) \tan f(x) dx = \frac{\sec f(x)}{f'(x)} + c$
$\int \csc x \cot x dx = -\csc x + c$	$\int \csc f(x) \cot f(x) dx = \frac{-\csc f(x)}{f'(x)} + c$
$\int \csc^2 x dx = -\cot x + c$	$\int \csc^2 f(x) dx = \frac{-\cot f(x)}{f'(x)} + c$

STANDARD FORM exponential	GENERAL FORM Where : $f(x)=ax+b$
$\int e^x dx = e^x + c$	$\int e^{f(x)} dx = \frac{e^{f(x)}}{f'(x)} + c$

STANDARD FORM Form 1/x	GENERAL FORM Where : $f(x)=ax+b$
$\int \frac{1}{x} dx = \ln\|x\| + c$	$\int \frac{1}{f(x)} dx = \frac{\ln\|f(x)\|}{f'(x)} + c$