Document No : UniKL MFI_SD_AC41
Revision No: 02
Effective Date: 01 December 2008

CONFIDENTIAL

SET A

UNIVERSITI KUALA LUMPUR
Malaysia France Institute

FINAL EXAMINATION
JULY 2010 SESSION

SUBJECT CODE	:	FKB 13102 / FKB 14102
SUBJECT TITLE	:	ENGINEERING MATHEMATICS 1
LEVEL	:	BACHELOR
TIME / DURATION	:	9.00am – 11.00am (2 HOURS)
DATE	:	12 NOVEMBER 2010

INSTRUCTIONS TO CANDIDATES

1. Please read the instructions given in the question paper CAREFULLY.
2. This question paper is printed on both sides of the paper.
3. Please write your answers on the answer booklet provided.
4. Answer should be written in blue or black ink except for sketching, graphic and illustration.
5. This question paper consists of SIX (6) questions. Answer four (4) questions only.
6. Answer all questions in English.
7. Fomulas are appended.

THERE ARE 6 PAGES OF QUESTIONS AND 7 PAGES OF APPENDIX, EXCLUDING THIS PAGE.

INSTRUCTION: Answer FOUR questions only (Total: 60 marks)

Please use the answer booklet provided.

Question 1

(a) If $A = \begin{pmatrix} -2 & a & -19 \\ b & -4 & 10 \\ 1 & c & 7 \end{pmatrix}$ and $\operatorname{adj}(A) = \begin{pmatrix} p & 1 & 4 \\ 3 & q & 1 \\ 1 & 2 & r \end{pmatrix}$, find a, b, c, p, q and r.

(6 marks)

(b) A square matrix that does not have an inverse is known as singular matrix. If M is the matrix $\begin{pmatrix} 3 & 1 & -3 \\ 1 & 2k & 1 \\ 0 & 2 & k \end{pmatrix}$

(i) Find the two values of k if M is a singular matrix.

(3 marks)

(ii) Using the **Cramer's Rule**, solve the equation $M\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3\frac{1}{2} \\ 5\frac{1}{2} \\ 5 \end{pmatrix}$ in the case when $k = 2$

(6 marks)

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(6 marks)

Question 2

(a) Given matrix $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a & b & c \end{pmatrix}$

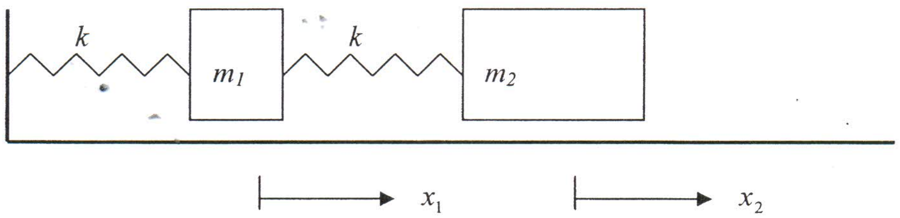
(i) Find the characteristic polynomial of the matrix A

(2 marks)

(ii) Can you find a $3\text{by}3$ matrix M whose characteristic polynomial is $-\lambda^3 + 17\lambda^2 - 5\lambda + \pi$?

(1 mark)

(b) Look at the spring-mass system as shown in the picture below.



Assume each of the two mass-displacements to be denoted by x_1 and x_2, and let us assume each spring has the same spring constant k. Then by applying Newton's 2nd and 3rd law of motion to develop a force-balance for each mass we have

$$m_1 \frac{d^2x_1}{dt^2} = -kx_1 + k(x_2 - x_1)$$

$$m_2 \frac{d^2x_2}{dt^2} = -k(x_2 - x_1)$$

From the vibration theory, the equations can be rewritten as $A\,x = \lambda x$

If $A = \begin{pmatrix} 2 & -3 \\ 4 & -5 \end{pmatrix}$

(i) Find $\det(A)$ and $-\text{tr}(A)$ **(2 marks)**

(ii) Show that the characteristic polynomial is $P(\lambda) = \lambda^2 - \text{tr}(A)\lambda + \det(A)$

(2 marks)

(iii) Find the eigenvalues of the matrix **(2 marks)**

(iv) Find the eigenvectors corresponding to the eigenvalues in (iii). **(6 marks)**

Question 3

(a) In control theory, it is often useful to work with complex numbers in the form $Z = e^{st}$ where s is the complex number $\sigma + j\omega$ and where t, σ and ω are real. The quantity ω is a frequency.

(i) Find the real and imaginary parts of Z

(3.5 marks)

(ii) Find $|Z|$

(0.5 mark)

(b) The complex numbers Z_1 and Z_2 satisfy the equation $Z^2 = 2 - j\,2\sqrt{3}$

(i) Express Z_1 and Z_2 in the form $a + jb$, where a and b are real numbers

(5 marks)

(ii) Represent Z_1 and Z_2 in an Argand Diagram

(2 marks)

(iii) For each of Z_1 and Z_2 find the modulus and the argument in radians.

(4 marks)

Question 4

(a) Consider the polynomial $P(Z) = Z^5 - Z^4 + 4Z^3 - 2Z^2 + 8$

Show that $Z_1 = 1 - j$ and $Z_2 = j2$ are the roots of $P(Z)$

(6 marks)

(b) A common requirement in control theory is to find the poles of a rational function, $G(s)$. The poles are the values of s that make the denominator zero.

(i) Find the poles of the denominator of $G(s) = \dfrac{3}{s\left(s^2 + 2s + 5\right)}$ in the Complex Domain.

(2 marks)

(ii) Hence, decompose $G(s)$ into partial fractions, also in the Complex Domain.

(Heaviside Method is recommended)

(7 marks)

Question 5

(a) If $y = UV$ where U and V are functions of x, then the derivative of y can be obtained by using the **PRODUCT RULE**, $\frac{dy}{dx} = V\frac{dU}{dx} + U\frac{dV}{dx}$

Given a function $y = e^{-2mx}\sin 4mx$ where m is a constant.

(i) Find the first derivative, $\frac{dy}{dx}$

(2 marks)

(ii) Find the second derivative, $\frac{d^2y}{dx^2}$

(3 marks)

(iii) Hence, show that $\frac{d^2y}{dx^2} + 4m\frac{dy}{dx} + 20m^2y = 0$

(3 marks)

(b) If $y = \frac{U}{V}$ where U and V are functions of x, then the derivative of y can be obtained by using the **QUOTIENT RULE**, $\frac{dy}{dx} = \frac{V\frac{dU}{dx} - U\frac{dV}{dx}}{V^2}$

(i) Differentiate $F(x) = \frac{\cosh x - 1}{\cosh x + 1}$

(3 marks)

(ii) Hence, find the derivative of $G(x) = \ln\left(\frac{\cosh x - 1}{\cosh x + 1}\right)$

(The following hyperbolic identity may be useful: $\cosh^2 x - \sinh^2 x = 1$)

(4 marks)

Question 6

(a) A vehicle of mass 500 kg, travelling on a horizontal surface, has its engine working at a constant rate of 10kW against a resisting force of $25v$ N, where v is the speed in ms^{-1}.

The time taken for the car to increase its speed from 5 ms^{-1} to 15 ms^{-1} is given by

$$T = \int_{5}^{15} \frac{20\,v}{400 - v^2}\,dv$$

Find T in seconds by applying the ***integration by substitution***

(5 marks)

(b) (i) By completing the square, prove that $x^2 + 6x + 16 = Z^2 + A^2$.

Determine Z and A

(2 marks)

(ii) Hence, determine $\int \frac{1}{2x^2 + 12x + 32}\,dx$

(3 marks)

(c) By using ***Integration by Parts***, determine $\int e^{3x} \sin x \, dx$

(5 marks)

END OF QUESTION

APPENDIX 1

Table of Differentiation

Trigonometric Functions - GENERAL FORM
$\frac{d}{dx}(\sin f(x)) = f'(x)\cos f(x)$
$\frac{d}{dx}(\cos f(x)) = -f'(x)\sin f(x)$
$\frac{d}{dx}(\tan f(x)) = f'(x)\sec^2 f(x)$
$\frac{d}{dx}(\csc f(x)) = -f'(x)\csc f(x)\cot f(x)$
$\frac{d}{dx}(\sec f(x)) = f'(x)\sec f(x)\tan f(x)$
$\frac{d}{dx}(\cot f(x)) = -f'(x)\csc^2 f(x)$

Exponential Function - GENERAL FORM
$\frac{d}{dx}\left(e^{f(x)}\right) = f'(x)\,e^{f(x)}$

Logarithmic Function - GENERAL FORM
$\frac{d}{dx}(\ln f(x)) = \frac{1}{f(x)}\,f'(x)$

APPENDIX 2

Derivatives of Inverse Trigonometric Functions
$\frac{d}{dx}\left(\sin^{-1} u\right)=\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$, $\lvert u \rvert<1$
$\frac{d}{dx}\left(\cos^{-1} u\right)=\frac{-1}{\sqrt{1-u^2}}\frac{du}{dx}$, $\lvert u \rvert<1$
$\frac{d}{dx}\left(\tan^{-1} u\right)=\frac{1}{1+u^2}\frac{du}{dx}$
$\frac{d}{dx}\left(\csc^{-1} u\right)=\frac{-1}{\lvert u \rvert\sqrt{u^2-1}}\frac{du}{dx}$, $\lvert u \rvert>1$
$\frac{d}{dx}\left(\sec^{-1} u\right)=\frac{1}{\lvert u \rvert\sqrt{u^2-1}}\frac{du}{dx}$, $\lvert u \rvert>1$
$\frac{d}{dx}\left(\cot^{-1} u\right)=\frac{-1}{1+u^2}\frac{du}{dx}$

Derivatives of Hyperbolic Functions
$\frac{d}{dx}\left(\sinh u\right)=\cosh u\frac{du}{dx}$
$\frac{d}{dx}\left(\cosh u\right)=\sinh u\frac{du}{dx}$
$\frac{d}{dx}\left(\tanh u\right)=\text{sech}^2 u\frac{du}{dx}$
$\frac{d}{dx}\left(\text{csch}\, u\right)=-\text{csch}\, u\coth u\frac{du}{dx}$
$\frac{d}{dx}\left(\text{sech}\, u\right)=-\text{sech}\, u\tanh u\frac{du}{dx}$
$\frac{d}{dx}\left(\coth u\right)=-\text{csch}^2 u\frac{du}{dx}$

APPENDIX 3

Derivatives of Inverse Hyperbolic Functions
$\frac{d}{dx}\left(\sinh^{-1} u\right) = \frac{1}{\sqrt{1+u^2}}\frac{du}{dx}$
$\frac{d}{dx}\left(\cosh^{-1} u\right) = \frac{1}{\sqrt{u^2-1}}\frac{du}{dx} \quad , \quad u > 1$
$\frac{d}{dx}\left(\tanh^{-1} u\right) = \frac{1}{1-u^2}\frac{du}{dx} \quad , \quad \lvert u \rvert < 1$
$\frac{d}{dx}\left(\operatorname{csch}^{-1} u\right) = \frac{-1}{\lvert u \rvert\sqrt{1+u^2}}\frac{du}{dx} \quad , \quad u \neq 0$
$\frac{d}{dx}\left(\operatorname{sech}^{-1} u\right) = \frac{-1}{u\sqrt{1-u^2}}\frac{du}{dx} \quad , \quad 0 < u < 1$
$\frac{d}{dx}\left(\coth^{-1} u\right) = \frac{1}{1-u^2}\frac{du}{dx} \quad , \quad \lvert u \rvert > 1$

APPENDIX 4

Trigonometric Identities and Formulas

FUNDAMENTAL IDENTITIES

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

FORMULAS FOR NEGATIVES

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta$$

$$\csc(-\theta) = -\csc\theta$$

$$\sec(-\theta) = \sec\theta$$

$$\cot(-\theta) = -\cot\theta$$

ADDITION FORMULAS

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

SUBTRACTION FORMULAS

$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

HALF-ANGLE FORMULAS

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$$

DOUBLE-ANGLE FORMULAS

$$\sin 2\theta = 2\sin\theta\sin\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$.......... = 1 - 2\sin^2\theta$$

$$.......... = 2\cos^2\theta - 1$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

PRODUCT-TO-SUM FORMULAS

$$\sin\alpha in\alpha c = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$

$$\cos\alpha os\alpha s = \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$$

$$\cos\alpha os\alpha c = \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$$

$$\sin\alpha in\alpha s = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$

SUM-TO-PRODUCT FORMULAS

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

APPENDIX 5

Integration of Inverse Trigonometric Functions
$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x \rvert < a$
$\int \frac{-1}{\sqrt{a^2 - x^2}}\,dx = \cos^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x \rvert < a$
$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
$\int \frac{-1}{\lvert x \rvert \sqrt{x^2 - a^2}}\,dx = \frac{1}{a}\csc^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x \rvert > a$
$\int \frac{1}{\lvert x \rvert \sqrt{x^2 - a^2}}\,dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x \rvert > a$
$\int \frac{-1}{a^2 + x^2}\,dx = \frac{1}{a}\cot^{-1}\left(\frac{x}{a}\right) + C$

Integration of Hyperbolic Functions Where $f(x) = ax + b$ and $f'(x) = a$
$\int \cosh f(x)\,dx = \frac{\sinh f(x)}{f'(x)} + C$
$\int \sinh f(x)\,dx = \frac{\cosh f(x)}{f'(x)} + C$
$\int \operatorname{sech}^2 f(x)\,dx = \frac{\tanh f(x)}{f'(x)} + C$
$\int \operatorname{csch}^2 f(x)\,dx = \frac{-\coth f(x)}{f'(x)} + C$
$\int \operatorname{sech} f(x) \tanh f(x)\,dx = \frac{-\operatorname{sech} f(x)}{f'(x)} + C$
$\int \operatorname{csch} f(x) \coth f(x)\,dx = \frac{-\operatorname{csch} f(x)}{f'(x)} + C$

APPENDIX 6

Integration of Inverse Hyperbolic Functions
$\int \frac{1}{\sqrt{a^2+x^2}}\,dx = \sinh^{-1}\left(\frac{x}{a}\right)+C \quad , \quad a>0$
$\int \frac{-1}{\sqrt{x^2-a^2}}\,dx = \cosh^{-1}\left(\frac{x}{a}\right)+C \quad , \quad x>a$
$\int \frac{1}{a^2-x^2}\,dx = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right)+C \quad , \quad \lvert x\rvert<a \\ \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right)+C \quad , \quad \lvert x\rvert>a \end{cases}$
$\int \frac{1}{x\sqrt{a^2+x^2}}\,dx = -\frac{1}{a}\operatorname{csch}^{-1}\left(\frac{x}{a}\right)+C \quad , \quad 0<x<a$
$\int \frac{1}{x\sqrt{a^2-x^2}}\,dx = -\frac{1}{a}\operatorname{sech}^{-1}\left(\frac{x}{a}\right)+C \quad , \quad 0<x<a$

APPENDIX 7

Table of Integration

Where $f(x) = ax + b$ and $f'(x) = a$

Trigonometric Functions - GENERAL FORM
$\int \cos f(x)dx = \frac{\sin f(x)}{f'(x)} + C$
$\int \sin f(x)dx = -\frac{\cos f(x)}{f'(x)} + C$
$\int \sec^2 f(x)dx = \frac{\tan f(x)}{f'(x)} + C$
$\int \csc^2 f(x)dx = -\frac{\cot f(x)}{f'(x)} + C$
$\int \sec f(x)\tan f(x)dx = \frac{\sec f(x)}{f'(x)} + C$
$\int \csc f(x)\cot f(x)dx = -\frac{\csc f(x)}{f'(x)} + C$

Exponential Function - GENERAL FORM
$\int e^{f(x)}\,dx = \frac{e^{f(x)}}{f'(x)} + C$

Logarithmic Function - GENERAL FORM
$\int \frac{1}{f(x)}dx = \frac{\ln\lvert f(x)\rvert}{f'(x)} + C$