SET A

UNIVERSITI KUALA LUMPUR
Malaysia France Institute

FINAL EXAMINATION
JANUARY 2011 SESSION

SUBJECT CODE	:	FKB 13102/FKB 14102
SUBJECT TITLE	:	ENGINEERING MATHEMATICS 1
LEVEL	:	BACHELOR
TIME / DURATION	:	12.30pm – 2.30pm (2 HOURS)
DATE	:	07 MAY 2011

INSTRUCTIONS TO CANDIDATES

1. Please read the instructions given in the question paper CAREFULLY.
2. Please write your answers on the answer booklet provided.
3. Answer should be written in blue or black ink except for sketching, graphic and illustration.
4. This question paper consists of TWO (2) sections. Section A and B. Answer all questions in Section A. For Section B, answer TWO (2) questions only.
5. Answer all questions in English.
7. Fomula is appended.

THERE ARE 6 PAGES OF QUESTIONS AND 3 PAGES OF APPENDIX, EXCLUDING THIS PAGE.

INSTRUCTION: Answer FOUR questions only (Total: 60 marks)

Please use the answer booklet provided.

Question 1

Matrix A is given as follows.

$$A = \begin{pmatrix} 2 & 1 & 3 \\ 4 & 2 & 5 \\ 5 & 3 & 6 \end{pmatrix}$$

(a) What is the size of matrix A ? **(1 mark)**

(b) What is the element a_{32} ? **(1 mark)**

(c) Calculate the minor for element 6. **(1 mark)**

(d) Calculate the cofactor for element in row 2 column 3. **(2 marks)**

(e) What are the elements in the leading diagonal of matrix A ? **(1 mark)**

(f) Show that $\det(A) = 1$ **(2 marks)**

(g) Find the adjoint matrix for matrix A , **(4 marks)**

(h) Using part (f) and (g) above, solve the following system of linear equations.

$$\begin{aligned} 2x + y + 3z &= 2 \\ 4x + 2y + 5z &= 1 \\ 5x + 3y + 6z &= 3 \end{aligned}$$

(3 marks)

Question 2

A matrix operator is defined as $A = \begin{pmatrix} 5 & 4 & 2 \\ 4 & 5 & 2 \\ 2 & 2 & 2 \end{pmatrix}$

(a) Calculate the characteristic polynomial $P(\lambda)$ for matrix A **(2 marks)**

(b) By using the calculator, find λ_1 , λ_2 and λ_3, the eigenvalues of matrix A **(1 mark)**

(c) Show that $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$ are the eigenvectors of matrix A and state the corresponding eigenvalues. **(2 marks)**

(d) Using the Gauss Elimination method, find the third eigenvector. **(5 marks)**

(e) **Without computation**, find a diagonal matrix D that is similar to matrix A and matrix P such that $P^{-1}AP = D$. Prove that the inverse of matrix P exist (without calculating the inverse itself) **(3 marks)**

The trace of an n-by-n square matrix A, denoted by $tr(A)$ is defined to be the sum of the elements on the leading diagonal.

(f) Show that $tr(A) = \lambda_1 + \lambda_2 + \lambda_3$ **(2 marks)**

Question 3

(a) Use implicit differentiation to find $\frac{dy}{dx}$ at point $(3,2)$ for $xy+2x^2=3y^2+12$

(4 marks)

(b) (i) Find the derivative of $f(x)=ax^2+bx+c$, where a, b and c are constants.

(1 mark)

(ii) The function $H(r)$ is defined as $H(r)=\frac{(r-2)^2}{(2r+1)^3}$. The derivative of this function is $\frac{dH}{dr}=\frac{2(r-2)(ar+b)}{(2r+1)^n}$. Determine the value of a, b and n.

(4 marks)

(iii) Find the third derivative of $g(x)=\sqrt{3-2x}$ **(2 marks)**

(c) Find $\int x\sqrt{2x+1}\,dx$ by Substitution **(4 marks)**

Question 4

(a) A rational function is defined as $\frac{x^3}{(x-1)(x-2)}$.

(i) Use the Euclidien Division to find the quotient and the remainder. Hence, write your result in the following form.

$$\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}$$

(3 marks)

(ii) Use the result found in (i) to decompose $\frac{x^3}{(x-1)(x-2)}$ into partial fractions.

(3 marks)

(iii) Hence, evaluate $\int_3^4 \frac{x^3}{(x-1)(x-2)}\,dx$ **(4 marks)**

(b) A function $G(t)$ is defined as $G(t) = \frac{e^t}{e^t+1}$

(i) Find $\int G(t)\,dt$ by Substitution **(3 marks)**

(ii) Show that $\frac{1}{e^t+1} = 1 - \frac{e^t}{e^t+1}$ **(1 mark)**

(iii) Hence, find $\int \frac{1}{e^t+1}\,dt$ **(1 mark)**

Question 5

A polynomial of degree 3 is defined by $P(Z) = Z^3 - 3Z^2 + 9Z + 13$

(a) Show that $P(2 - i3) = 0$ and give your conclusion. **(3 marks)**

(b) Factorize $P(Z)$ completely. **(4 mark)**

(c) Find all the roots when $P(Z) = 0$. **(1 mark)**

A rational fraction is defined by $F(Z) = \dfrac{Z+1}{Z^3 - 3Z^2 + 9Z + 13}$

(d) Use the result found in (b) to decompose $F(Z)$ into partial fractions. The Heaviside Method is recommended. **(7 marks)**

Question 6

(a) (i) Simplify $a = \dfrac{3 - i2}{\frac{1}{2} - i\frac{5}{2}}$ into $x + iy$ form **(2 marks)**

(ii) Express the complex number, a found in part (a) into the **exponential form**, $re^{j\theta}$ **(2 marks)**

(iii) Using the result found in part (b), solve the equation $Z^4 = a$ expressing the solution in exponential form. Show the results on the Argand Diagram.

(4 marks)

(b) A second degree equation is defined as $iZ^2 + (1 - i5)Z - 1 + i8 = 0$

(i) Show that the discriminant for the given equation is $\delta^2 = 8 - i6$

(1 mark)

(ii) Find the roots of the discriminant, δ_1 and δ_2 in part (a) **(4 marks)**

(iii) Solve the given equation, where $Z_i = \dfrac{-b + \delta_i}{2a}$ **(2 marks)**

END OF QUESTION

APPENDIX 1 - Trigonometric Identities and Formulas

Fundamental Identities

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Formulas For Negatives

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta$$

$$\csc(-\theta) = -\csc\theta$$

$$\sec(-\theta) = \sec\theta$$

$$\cot(-\theta) = -\cot\theta$$

Addition Formulas

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

Subtraction Formulas

$$\sin(A - B) = \sin A\cos B - \cos A\sin B$$

$$\cos(A - B) = \cos A\cos B + \sin A\sin B$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

Half-Angle Formulas

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$$

Double-Angle Formulas

$$\sin 2\theta = 2\sin\theta\sin\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$.......... = 1 - 2\sin^2\theta$$

$$.......... = 2\cos^2\theta - 1$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

Product-To-Sum Formulas

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$

$$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$$

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$$

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$

Sum-To-Product Formulas

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

APPENDIX 2 – Table of Differentiation

Trigonometric Functions

$$\frac{d}{dx}(\sin f(x)) = f'(x)\cos f(x)$$

$$\frac{d}{dx}(\cos f(x)) = -f'(x)\sin f(x)$$

$$\frac{d}{dx}(\tan f(x)) = f'(x)\sec^2 f(x)$$

$$\frac{d}{dx}(\csc f(x)) = -f'(x)\csc f(x)\cot f(x)$$

$$\frac{d}{dx}(\sec f(x)) = f'(x)\sec f(x)\tan f(x)$$

$$\frac{d}{dx}(\cot f(x)) = -f'(x)\csc^2 f(x)$$

Inverse Trigonometric Functions

$$\frac{d}{dx}(\sin^{-1}U) = \frac{1}{\sqrt{1-U^2}}\frac{dU}{dx}, \quad |U|<1$$

$$\frac{d}{dx}(\cos^{-1}U) = \frac{-1}{\sqrt{1-U^2}}\frac{dU}{dx}, \quad |U|<1$$

$$\frac{d}{dx}(\tan^{-1}U) = \frac{1}{1+U^2}\frac{dU}{dx}$$

$$\frac{d}{dx}(\csc^{-1}U) = \frac{-1}{|U|\sqrt{U^2-1}}\frac{dU}{dx}, \quad |U|>1$$

$$\frac{d}{dx}(\sec^{-1}U) = \frac{1}{|U|\sqrt{U^2-1}}\frac{dU}{dx}, \quad |U|>1$$

$$\frac{d}{dx}(\cot^{-1}U) = \frac{-1}{1+U^2}\frac{dU}{dx}$$

Hyperbolic Functions

$$\frac{d}{dx}(\sinh U) = \cosh U\frac{dU}{dx}$$

$$\frac{d}{dx}(\cosh U) = \sinh U\frac{dU}{dx}$$

$$\frac{d}{dx}(\tanh U) = \operatorname{sech}^2 U\frac{dU}{dx}$$

$$\frac{d}{dx}(\operatorname{csch} U) = -\operatorname{csch} U\coth U\frac{dU}{dx}$$

$$\frac{d}{dx}(\operatorname{sech} U) = -\operatorname{sech} U\tanh U\frac{dU}{dx}$$

$$\frac{d}{dx}(\coth U) = -\operatorname{csch}^2 U\frac{dU}{dx}$$

Inverse Hyperbolic Functions

$$\frac{d}{dx}(\sinh^{-1} U) = \frac{1}{\sqrt{1+U^2}}\frac{dU}{dx}$$

$$\frac{d}{dx}(\cosh^{-1} U) = \frac{1}{\sqrt{U^2-1}}\frac{dU}{dx}, \quad U>1$$

$$\frac{d}{dx}(\tanh^{-1} U) = \frac{1}{1-U^2}\frac{dU}{dx}, \quad |U|<1$$

$$\frac{d}{dx}(\operatorname{csch}^{-1} U) = \frac{-1}{|U|\sqrt{1+U^2}}\frac{dU}{dx}, \quad U\neq 0$$

$$\frac{d}{dx}(\operatorname{sech}^{-1} U) = \frac{-1}{U\sqrt{1-U^2}}\frac{dU}{dx}, \quad 0<U<1$$

$$\frac{d}{dx}(\coth^{-1} U) = \frac{1}{1-U^2}\frac{dU}{dx}, \quad |U|>1$$

Exponential Function

$$\frac{d}{dx}\left(e^{f(x)}\right) = f'(x)e^{f(x)}$$

Natural Logarithmic Function

$$\frac{d}{dx}(\ln f(x)) = \frac{1}{f(x)}f'(x)$$

APPENDIX 3 – Table of Integration

Trigonometric Functions Where $f(x)=ax+b$
$\int \cos f(x)dx = \frac{\sin f(x)}{f'(x)} + C$
$\int \sin f(x)dx = -\frac{\cos f(x)}{f'(x)} + C$
$\int \sec^2 f(x)dx = \frac{\tan f(x)}{f'(x)} + C$
$\int \csc^2 f(x)dx = -\frac{\cot f(x)}{f'(x)} + C$
$\int \sec f(x)\tan f(x)dx = \frac{\sec f(x)}{f'(x)} + C$
$\int \csc f(x)\cot f(x)dx = -\frac{\csc f(x)}{f'(x)} + C$

Inverse Trigonometric Functions
$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x \rvert < a$
$\int \frac{-1}{\sqrt{a^2-x^2}}dx = \cos^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x \rvert < a$
$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
$\int \frac{-1}{\lvert x \rvert\sqrt{x^2-a^2}}dx = \frac{1}{a}\csc^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x \rvert > a$
$\int \frac{1}{\lvert x \rvert\sqrt{x^2-a^2}}dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x \rvert > a$
$\int \frac{-1}{a^2+x^2}dx = \frac{1}{a}\cot^{-1}\left(\frac{x}{a}\right) + C$

Hyperbolic Functions Where $f(x)=ax+b$
$\int \cosh f(x)dx = \frac{\sinh f(x)}{f'(x)} + C$
$\int \sinh f(x)dx = \frac{\cosh f(x)}{f'(x)} + C$
$\int \operatorname{sech}^2 f(x)dx = \frac{\tanh f(x)}{f'(x)} + C$
$\int \operatorname{csch}^2 f(x)dx = -\frac{\coth f(x)}{f'(x)} + C$
$\int \operatorname{sech} f(x)\tanh f(x)dx = \frac{-\operatorname{sech} f(x)}{f'(x)} + C$
$\int \operatorname{csch} f(x)\coth f(x)dx = -\frac{\operatorname{csch} f(x)}{f'(x)} + C$

Inverse Hyperbolic Functions
$\int \frac{1}{\sqrt{a^2+x^2}}dx = \sinh^{-1}\left(\frac{x}{a}\right) + C \quad , \quad a > 0$
$\int \frac{-1}{\sqrt{x^2-a^2}}dx = \cosh^{-1}\left(\frac{x}{a}\right) + C \quad , \quad x > a$
$\int \frac{1}{a^2-x^2}dx = \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x \rvert < a$
$\int \frac{1}{a^2-x^2}dx = \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right) + C \quad , \quad \lvert x \rvert > a$
$\int \frac{1}{x\sqrt{a^2+x^2}}dx = -\frac{1}{a}\operatorname{csch}^{-1}\left(\frac{x}{a}\right) + C \quad , \quad 0 < x < a$
$\int \frac{1}{x\sqrt{a^2-x^2}}dx = -\frac{1}{a}\operatorname{sech}^{-1}\left(\frac{x}{a}\right) + C \quad , \quad 0 < x < a$

Exponential Function Where $f(x)=ax+b$
$\int e^{f(x)}dx = \frac{e^{f(x)}}{f'(x)} + C$

Form $\frac{1}{f(x)}$, where $f(x)=ax+b$
$\int \frac{1}{f(x)}dx = \frac{\ln\lvert f(x) \rvert}{f'(x)} + C$