



UNIVERSITI KUALA LUMPUR
MALAYSIAN INSTITUTE OF MARINE ENGINEERING TECHNOLOGY

FINAL EXAMINATION
SEPTEMBER 2016 SEMESTER

COURSE CODE : LGB 22003
COURSE NAME : MATHEMATICS 3
PROGRAMME NAME : BACHELOR OF MARINE ELECTRICAL AND
(FOR MPU: PROGRAMME LEVEL) ELECTRONICS
DATE : 19 JANUARY 2016
TIME : 09.00 AM – 12.00 PM
DURATION : 3 HOURS

INSTRUCTIONS TO CANDIDATES

1. Please **CAREFULLY** read the instructions given in the question paper.
2. This question paper has information printed on both sides of the paper.
3. This question paper consists of **TWO (2)** sections; Section A and Section B.
4. Answer **ALL** questions in Section A. For Section B, answer **THREE (3)** questions only.
5. Please write your answers on the answer booklet provided.
6. Answer all questions in English language **ONLY**.
7. Formulas has been appended for your reference.

THERE ARE 6 PAGES OF QUESTIONS, INCLUDING THIS PAGE.

SECTION A (Total: 40 marks)

INSTRUCTION: Answer ALL FIVE questions.

Please use the answer booklet provided.

Question 1

(a) Solve the second order partial derivatives of $f(x, y, z) = 6z^4x^2y^3 + x^4yz$ for:

i. $\frac{d^2f}{dzdy}$

(2 marks)

ii. $\frac{d^2f}{dxdz}$

(2 marks)

(b) Determine the third order partial derivatives of $f(x, y, z) = zy^2e^x + z^2y$ for f_{xyz} .

(4 marks)

Question 2

By using definition of convolution, solve the convolution the following functions:

(a) $(g(t) = 5e^{6t}) * (f(t) = 1)$

(4 marks)

(b) $(g(t) = 5e^{6t}) * (f(t) = e^{-2t})$

(4 marks)

Question 3

Prove that for any two complex numbers z and w , the following are true.

(a) $z\bar{z} = a^2 + b^2$

(4 marks)

(b) $\overline{z + w} = \bar{z} + \bar{w}$

(4 marks)

Question 4

Obtain the unit step response of the system specified by the second order difference equation

$y_n - \frac{3}{4}y_{n-1} + \frac{1}{8}y_{n-2} = x_n$ by using z transform.

(8 marks)

Question 5

(a) Determine the Fourier Transform $Y(\omega)$ if given $y(t) = 25\text{rect}\left(\frac{(t-4)}{10}\right)$

(4 marks)

(b) Given Fourier Transform $Y(\omega) = \frac{1+j\omega}{2+j\omega}$, determine the inverse of Fourier Transform $Y(\omega)$.

(4 marks)

SECTION B (Total: 80marks)

INSTRUCTION: Total questions are FIVE but answer only THREE questions.

Please use the answer booklet provided.

Question 6

- (a) Calculate the rate is the volume of a box changing if its length is 8 ft and increasing toward time at 3 ft/s, its width is 6ft and increasing towards time at 2 ft/s and its height is 4 ft and increasing at 1 ft/s by using partial differentiation.

(8 marks)

- (b) Given that $z = e^{xy}$, $x = 2u + v$ and $y = \frac{u}{v}$.

- i. Show the tree diagram for the formulas of differentiation

(4 marks)

- ii. Differentiate z with respect to u and v .

(8 marks)

Question 7

Based on Figure 1,

- i. Determine the range for convolution process

(3 marks)

- i. Evaluate the convolution process between the signals. Use Appendix 1 for the answers.[Hint: Use $f(t)$ as moving signal]

(13 marks)

- iii. Summarize the result of convolution and sketch the third signal, $y(t)$

(4 marks)

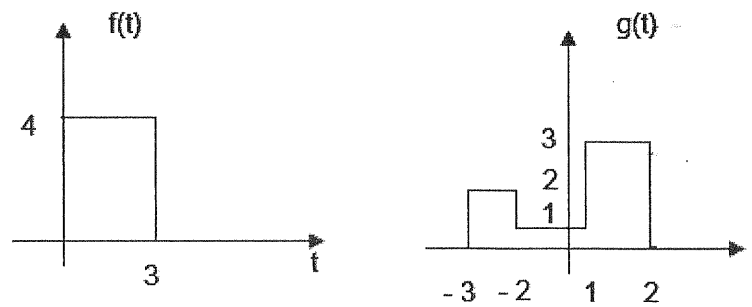


Figure 1

Question 8

- (a) Verify that the function $f(z) = e^{\alpha z}$ satisfies the Cauchy-Riemann equations and differentiate $f(z)$.

(8 marks)

- (b) Determine the contour integral $\oint_C \frac{1}{(z^2 + 2z + 2)(z^3)} dz$ if C is:

- i. the circle $|z| = 0.5$
- ii. the circle $|z| = 4$

(12 marks)

Question 9

Given $F(z) = \frac{0.5z}{(z - 0.5)(z - 0.7)}$, determine the sampled time function $f(t)$ by using:

- (a) partial fraction expansion

(10 marks)

- (b) long division

(10 marks)

Question 10

- (a) Given the signal of response below, determine the Fourier Transform of the signal

$$h(t) = \begin{cases} A & -2 \leq t < 0 \\ B & 0 \leq t < 2 \\ 0 & \text{otherwise} \end{cases}$$

(10 marks)

- (b) Using the continuous Fourier series to approximate the following periodic function ($T = 2\pi$ seconds) for $k = 1$ shown in Figure 2.

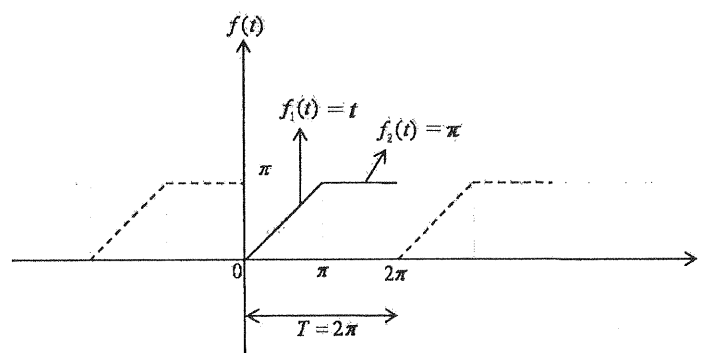


Figure 2

(10 marks)

END OF QUESTIONS PAPER

TRIGONOMETRY IDENTITIES

FUNDAMENTAL IDENTITIES	FORMULAS FOR NEGATIVES
$\csc \theta = \frac{1}{\sin \theta}$	$\sin(-\theta) = -\sin \theta$
$\sec \theta = \frac{1}{\cos \theta}$	$\cos(-\theta) = \cos \theta$
$\cot \theta = \frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta}$	$\tan(-\theta) = -\tan \theta$
$\cos^2 \theta = \frac{1}{2}(1 + \cos 2\theta)$	$\csc(-\theta) = -\csc \theta$
$\sin^2 \theta + \cos^2 \theta = 1$	$\sec(-\theta) = \sec \theta$
$1 + \tan^2 \theta = \sec^2 \theta$	$\cot(-\theta) = -\cot \theta$
$1 + \cot^2 \theta = \csc^2 \theta$	$\sin^2 \theta = \frac{1}{2}(1 - \cos 2\theta)$

ADDITION FORMULAS	SUBTRACTION FORMULAS
$\sin(A + B) = \sin A \cos B + \cos A \sin B$	$\sin(A - B) = \sin A \cos B - \cos A \sin B$
$\cos(A + B) = \cos A \cos B - \sin A \sin B$	$\cos(A - B) = \cos A \cos B + \sin A \sin B$
$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$	$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

HALF-ANGLE FORMULAS	DOUBLE-ANGLE FORMULAS
$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$	$\sin 2\theta = 2 \sin \theta \cos \theta$
$\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$	$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$ = $1 - 2 \sin^2 \theta$ = $2 \cos^2 \theta - 1$
$\tan \frac{\theta}{2} = \frac{1 - \cos \theta}{\sin \theta} = \frac{\sin \theta}{1 + \cos \theta}$	$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$

PRODUCT-TO-SUM FORMULAS	SUM-TO-PRODUCT FORMULAS
$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$	$\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$
$\cos \alpha \sin \beta = \frac{1}{2} [\sin(\alpha + \beta) - \sin(\alpha - \beta)]$	$\sin \alpha - \sin \beta = 2 \cos \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$
$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha + \beta) + \cos(\alpha - \beta)]$	$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$

DIFFERENTIATION

STANDARD FORM	GENERAL FORM
$\frac{d}{dx}(\sin x) = \cos x$	$\frac{d}{dx}(\sin f(x)) = f'(x) \cos f(x)$
$\frac{d}{dx}(\cos x) = -\sin x$	$\frac{d}{dx}(\cos f(x)) = -f'(x) \sin f(x)$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\frac{d}{dx}(\tan f(x)) = f'(x) \sec^2 f(x)$
$\frac{d}{dx}(\csc x) = -\csc x \cot x$	$\frac{d}{dx}(\csc f(x)) = -f'(x) \csc f(x) \cot f(x)$
$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\frac{d}{dx}(\sec f(x)) = f'(x) \sec f(x) \tan f(x)$
$\frac{d}{dx}(\cot x) = -\csc^2 x$	$\frac{d}{dx}(\cot f(x)) = -f'(x) \csc^2 f(x)$

EXPONENTIAL FUNCTION

STANDARD FORM	GENERAL FORM
$\frac{d}{dx} e^x = e^x$	$\frac{d}{dx} e^{f(x)} = f'(x) e^{f(x)}$

LOGARITHMIC FUNCTION

STANDARD FORM	GENERAL FORM
$\frac{d}{dx} \ln x = \frac{1}{x}$	$\frac{d}{dx} \ln f(x) = \frac{f'(x)}{f(x)}$

INTEGRATION

STANDARD FORM	GENERAL FORM Where : $f(x) = ax + b$
$\int \cos x \, dx = \sin x + c$	$\int \cos f(x) \, dx = \frac{\sin f(x)}{f'(x)} + c$
$\int \sin x \, dx = -\cos x + c$	$\int \sin f(x) \, dx = \frac{-\cos f(x)}{f'(x)} + c$
$\int \sec^2 x \, dx = \tan x + c$	$\int \sec^2 f(x) \, dx = \frac{\tan f(x)}{f'(x)} + c$
$\int \sec x \tan x \, dx = \sec x + c$	$\int \sec f(x) \tan f(x) \, dx = \frac{\sec f(x)}{f'(x)} + c$
$\int \csc x \cot x \, dx = -\csc x + c$	$\int \csc f(x) \cot f(x) \, dx = \frac{-\csc f(x)}{f'(x)} + c$
$\int \csc^2 x \, dx = -\cot x + c$	$\int \csc^2 f(x) \, dx = \frac{-\cot f(x)}{f'(x)} + c$
$\int \tan x \, dx = \ln \sec x + c$	$\int \tan f(x) \, dx = \frac{\ln \sec f(x) }{f'(x)} + c$
$\int \sec x \, dx = \ln \sec x + \tan x + c$	$\int \sec f(x) \, dx = \frac{\ln \sec f(x) + \tan f(x) }{f'(x)} + c$
$\int \cot x \, dx = \ln \sin x + c$	$\int \cot f(x) \, dx = \frac{\ln \sin f(x) }{f'(x)} + c$
$\int \csc x \, dx = -\ln \csc x + \cot x + c$	$\int \csc f(x) \, dx = \frac{-\ln \csc f(x) + \cot f(x) }{f'(x)} + c$

EXPONENTIAL FUNCTION

STANDARD FORM	GENERAL FORM Where : $f(x) = ax + b$
$\int e^x \, dx = e^x + c$	$\int e^{f(x)} \, dx = \frac{e^{f(x)}}{f'(x)} + c$

LOGARITHMIC FUNCTION

STANDARD FORM	GENERAL FORM Where : $f(x) = ax + b$
$\int \frac{1}{x} \, dx = \ln x + c$	$\int \frac{1}{f(x)} \, dx = \frac{\ln f(x) }{f'(x)} + c$

LAPLACE TRANSFORM

	$f(t)$	$F(s) = \mathcal{L}[f(t)](s)$
1.	1	$\frac{1}{s}$
2.	t	$\frac{1}{s^2}$
3.	$t^n (n = 1, 2, 3, \dots)$	$\frac{n!}{s^{n+1}}$
4.	$\frac{1}{\sqrt{t}}$	$\sqrt{\frac{\pi}{s}}$
5.	e^{at}	$\frac{1}{s-a}$
6.	te^{at}	$\frac{1}{(s-a)^2}$
7.	$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
8.	$\frac{1}{a-b}(e^{at} - e^{bt})$	$\frac{1}{(s-a)(s-b)}$
9.	$\frac{1}{a-b}(ae^{at} - be^{bt})$	$\frac{s}{(s-a)(s-b)}$
10.	$\frac{(c-b)e^{at} + (a-c)e^{bt} + (b-a)e^{ct}}{(a-b)(b-c)(c-a)}$	$\frac{1}{(s-a)(s-b)(s-c)}$
11.	$\sin(at)$	$\frac{a}{s^2 + a^2}$
12.	$\cos(at)$	$\frac{s}{s^2 + a^2}$
13.	$1 - \cos(at)$	$\frac{a^2}{s(s^2 + a^2)}$
14.	$at - \sin(at)$	$\frac{a^3}{s^2(s^2 + a^2)}$
15.	$\sin(at) - at \cos(at)$	$\frac{2a^3}{(s^2 + a^2)^2}$
16.	$\sin(at) + at \cos(at)$	$\frac{2as^2}{(s^2 + a^2)^2}$
17.	$t \sin(at)$	$\frac{2as}{(s^2 + a^2)^2}$
18.	$t \cos(at)$	$\frac{(s^2 - a^2)}{(s^2 + a^2)^2}$
19.	$\frac{\cos(at) - \cos(bt)}{(b-a)(b+a)}$	$\frac{s}{(s^2 + a^2)(s^2 + b^2)}$
20.	$e^{at} \sin(bt)$	$\frac{b}{(s-a)^2 + b^2}$
21.	$e^{at} \cos(bt)$	$\frac{s-a}{(s-a)^2 + b^2}$
22.	$\sinh(at)$	$\frac{a}{s^2 - a^2}$
23.	$\cosh(at)$	$\frac{s}{s^2 - a^2}$
24.	$\sin(at)\cosh(at) - \cos(at)\sinh(at)$	$\frac{4a^3}{s^4 + 4a^4}$
25.	$\sin(at)\sinh(at)$	$\frac{2a^2 s}{s^4 + 4a^4}$

	$f(t)$	$F(s) = \mathcal{L}[f(t)](s)$
26.	$\sinh(at) - \sin(at)$	$\frac{2a^3}{s^4 - a^4}$
27.	$\cosh(at) - \cos(at)$	$\frac{2a^2 s}{s^4 - a^4}$
28.	$\frac{1}{\sqrt{\pi t}} e^{at} (1 + 2at)$	$\frac{s}{(s-a)^{3/2}}$
29.	$J_0(at)$	$\frac{1}{\sqrt{s^2 + a^2}}$
30.	$J_n(at)$	$\frac{1}{a^n} \frac{(\sqrt{s^2 + a^2} - s)^n}{\sqrt{s^2 + a^2}}$
31.	$J_0(2\sqrt{at})$	$\frac{1}{s} e^{-a/s}$
32.	$\frac{1}{t} \sin(at)$	$\tan^{-1}\left(\frac{a}{s}\right)$
33.	$\frac{2}{t} [1 - \cos(at)]$	$\ln\left(\frac{s^2 + a^2}{s^2}\right)$
34.	$\frac{2}{t} [1 - \cosh(at)]$	$\ln\left(\frac{s^2 - a^2}{s^2}\right)$
35.	$\frac{1}{\sqrt{\pi t}} - a e^{a^2 t} \operatorname{erfc}\left(\frac{a}{\sqrt{t}}\right)$	$\frac{1}{\sqrt{s+a}}$
36.	$\frac{1}{\sqrt{\pi t}} + a e^{a^2 t} \operatorname{erf}\left(\frac{a}{\sqrt{t}}\right)$	$\frac{\sqrt{s}}{s-a^2}$
37.	$e^{a^2 t} \operatorname{erf}(a\sqrt{t})$	$\frac{a}{\sqrt{s}(s-a^2)}$
38.	$e^{a^2 t} \operatorname{erfc}(a\sqrt{t})$	$\frac{1}{\sqrt{s}(\sqrt{s+a})}$
39.	$\operatorname{erfc}\left(\frac{a}{2\sqrt{t}}\right)$	$\frac{1}{s} e^{-a\sqrt{s}}$
40.	$\frac{1}{\sqrt{\pi t}} e^{-a^2/4t}$	$\frac{1}{\sqrt{s}} e^{-a\sqrt{s}}$
41.	$\frac{1}{\sqrt{\pi(t+a)}}$	$\frac{1}{\sqrt{s}} e^{as} \operatorname{erfc}(\sqrt{as})$
42.	$\frac{1}{\pi} \sin(2a\sqrt{t})$	$\operatorname{erf}\left(\frac{a}{\sqrt{s}}\right)$
43.	$f\left(\frac{t}{a}\right)$	$aF(as)$
44.	$e^{bt/a} f\left(\frac{t}{a}\right)$	$aF(as-b)$
45.	$\delta_\epsilon(t)$	$\frac{e^{-\epsilon s}(1 - e^{-\epsilon s})}{\epsilon s}$
46.	$\delta(t-a)$	e^{-as}
47.	$L_n(t)$ (Laguerre polynomial)	$\frac{1}{s} \left(\frac{s-1}{s}\right)^n$

FOURIER TRANSFORM

Entry	$x(t)$	$X(f)$	$X(\omega)$
1	$\delta(t)$	1	1
2	$\text{rect}(t)$	$\text{sinc}(f)$	$\text{sinc}\left(\frac{\omega}{2\pi}\right)$
3	$\text{tri}(t)$	$\text{sinc}^2(f)$	$\text{sinc}^2\left(\frac{\omega}{2\pi}\right)$
4	$\text{sinc}(t)$	$\text{rect}(f)$	$\text{rect}\left(\frac{\omega}{2\pi}\right)$
5	$\cos(2\pi\alpha t)$	$0.5[\delta(f + \alpha) + \delta(f - \alpha)]$	$\pi[\delta(\omega + 2\pi\alpha) + \delta(\omega - 2\pi\alpha)]$
6	$\sin(2\pi\alpha t)$	$j0.5[\delta(f + \alpha) - \delta(f - \alpha)]$	$j\pi[\delta(\omega + 2\pi\alpha) - \delta(\omega - 2\pi\alpha)]$
7	$e^{-\alpha t}u(t)$	$\frac{1}{\alpha + j2\pi f}$	$\frac{1}{\alpha + j\omega}$
8	$te^{-\alpha t}u(t)$	$\frac{1}{(\alpha + j2\pi f)^2}$	$\frac{1}{(\alpha + j\omega)^2}$
9	$e^{-\alpha t }$	$\frac{2\alpha}{\alpha^2 + 4\pi^2 f^2}$	$\frac{2\alpha}{\alpha^2 + \omega^2}$
10	$e^{-\pi t^2}$	$e^{-\pi f^2}$	$e^{-\omega^2/4\pi}$
11	$\text{sgn}(t)$	$\frac{1}{j\pi f}$	$\frac{2}{j\omega}$
12	$u(t)$	$0.5\delta(f) + \frac{1}{j2\pi f}$	$\pi\delta(\omega) + \frac{1}{j\omega}$
13	$e^{-\alpha t} \cos(2\pi\beta t)u(t)$	$\frac{\alpha + j2\pi f}{(\alpha + j2\pi f)^2 + (2\pi\beta)^2}$	$\frac{\alpha + j\omega}{(\alpha + j\omega)^2 + (2\pi\beta)^2}$
14	$e^{-\alpha t} \sin(2\pi\beta t)u(t)$	$\frac{2\pi\beta}{(\alpha + j2\pi f)^2 + (2\pi\beta)^2}$	$\frac{2\pi\beta}{(\alpha + j\omega)^2 + (2\pi\beta)^2}$
15	$\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$\frac{1}{T} \sum_{k=-\infty}^{\infty} \delta\left(f - \frac{k}{T}\right)$	$\frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{T}\right)$
16	$x_p(t) = \sum_{k=-\infty}^{\infty} X[k]e^{j2\pi k f_0 t}$	$\sum_{k=-\infty}^{\infty} X[k]\delta(f - kf_0)$	$\sum_{k=-\infty}^{\infty} 2\pi X[k]\delta(\omega - k\omega_0)$

Property	$x(t)$	$X(f)$	$X(\omega)$
Similarity	$X(t)$	$x(-f)$	$2\pi x(-\omega)$
Time Scaling	$x(\alpha t)$	$\frac{1}{ \alpha } X\left(\frac{f}{\alpha}\right)$	$\frac{1}{ \alpha } X\left(\frac{\omega}{\alpha}\right)$
Folding	$x(-t)$	$X(-f)$	$X(-\omega)$
Time Shift	$x(t - \alpha)$	$e^{-j2\pi f \alpha} X(f)$	$e^{-j\omega \alpha} X(\omega)$
Frequency Shift	$e^{j2\pi \alpha t} x(t)$	$X(f - \alpha)$	$X(\omega - 2\pi \alpha)$
Convolution	$x(t) * h(t)$	$X(f)H(f)$	$X(\omega)H(\omega)$
Multiplication	$x(t)h(t)$	$X(f) * H(f)$	$\frac{1}{2\pi} X(\omega) * H(\omega)$
Modulation	$x(t)\cos(2\pi \alpha t)$	$0.5[X(f + \alpha) + X(f - \alpha)]$	$0.5[X(\omega + 2\pi \alpha) + X(\omega - 2\pi \alpha)]$
Derivative	$x'(t)$	$j2\pi f X(f)$	$j\omega X(\omega)$
Times- t	$-j2\pi t x(t)$	$X'(f)$	$2\pi X'(\omega)$
Integration	$\int_{-\infty}^t x(t) dt$	$\frac{1}{j2\pi f} X(f) + 0.5X(0)\delta(f)$	$\frac{1}{j\omega} X(\omega) + \pi X(0)\delta(\omega)$
Conjugation	$x^*(t)$	$X^*(-f)$	$X^*(-\omega)$
Correlation	$x(t) ** y(t)$	$X(f)Y^*(f)$	$X(\omega)Y^*(\omega)$
Autocorrelation	$x(t) ** x(t)$	$X(f)X^*(f) = X(f) ^2$	$X(\omega)X^*(\omega) = X(\omega) ^2$

Fourier Transform Theorems

Central ordinates	$x(0) = \int_{-\infty}^{\infty} X(f) df = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) d\omega$	$X(0) = \int_{-\infty}^{\infty} x(t) dt$
Parseval's theorem	$E = \int_{-\infty}^{\infty} x^2(t) dt = \int_{-\infty}^{\infty} X(f) ^2 df = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) ^2 d\omega$	
Plancherel's theorem	$\int_{-\infty}^{\infty} x(t)y^*(t) dt = \int_{-\infty}^{\infty} X(f)Y^*(f) df = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)Y^*(\omega) d\omega$	

COMPLEX ANALYSIS

$$\int_{\Gamma} f(z) dz = \int_a^b f(\Gamma(t)) \Gamma'(t) dt$$

$$\oint_C \frac{f(z)}{z - z_0} dz = 2\pi i f(z_0)$$

Residue at pole $z = \lim_{z \rightarrow z_0} [(z - z_0) f(z)]$

Residue at pole $z = \frac{1}{(m-1)!} \lim_{z \rightarrow z_0} \left\{ \frac{d^{m-1}}{dz^{m-1}} [(z - z_0)^m f(z)] \right\}$

$$\oint_C f(z) dz = 2\pi i [\text{sum of residue of } f(z) \text{ at poles inside } C]$$

EULER'S FORMULAS

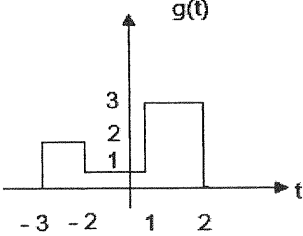
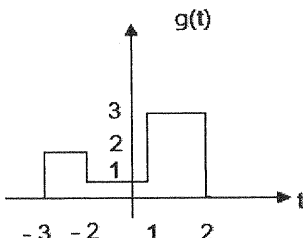
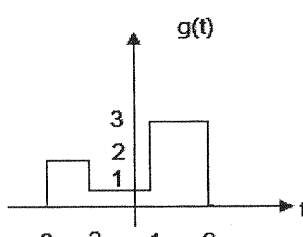
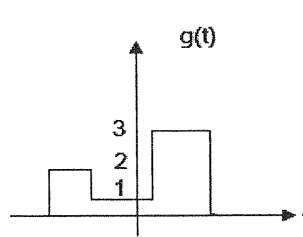
$$\cos x = \operatorname{Re}(e^{ix}) = \frac{e^{ix} + e^{-ix}}{2}$$

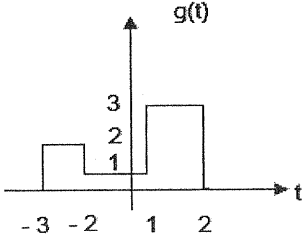
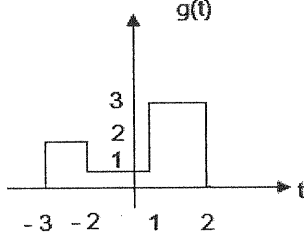
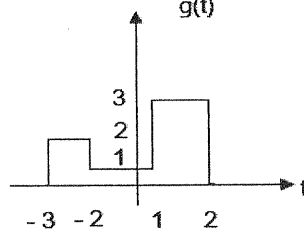
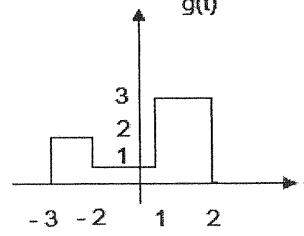
$$\sin x = \operatorname{Im}(e^{ix}) = \frac{e^{ix} - e^{-ix}}{2i}$$

TABLE OF Z-TRANSFORM

$x(t)$	$X(s)$	$X(z)$	$f(kT)$
1. $\delta(t) = \begin{cases} 1 & t=0, \\ 0 & t=kT, k \neq 0 \end{cases} \textcircled{1}$	1	1	$\textcircled{1}$
2. $\delta(t - kT) = \begin{cases} 1 & t=kT, \\ 0 & t \neq kT \end{cases} \textcircled{2}$	e^{-kTs}	z^{-k}	$\textcircled{2}$
3. $u(t)$, unit step	$1/s$	$\frac{z}{z-1}$	$u(kT)$
4. t	$1/s^2$	$\frac{Tz}{(z-1)^2}$	kT
5. t^2	$2/s^3$	$\frac{T^2 z(z+1)}{(z-1)^3}$	$(kT)^2$
6. e^{-at}	$\frac{1}{s+a}$	$\frac{z}{z-e^{-aT}}$	e^{-akT}
7. $1 - e^{-at}$	$\frac{a}{s(s+a)}$	$\frac{(1-e^{-aT})z}{(z-1)(z-e^{-aT})}$	—
8. te^{-at}	$\frac{1}{(s+a)^2}$	$\frac{Tze^{-aT}}{(z-e^{-aT})^2}$	$(kT)e^{-akT}$
9. $t^2 e^{-at}$	$\frac{2}{(s+a)^3}$	$\frac{T^2 e^{-aT} z(z+e^{-aT})}{(z-e^{-aT})^3}$	$(kT)^2 e^{-akT}$
10. $be^{-bt} - ae^{-at}$	$\frac{(b-a)s}{(s+a)(s+b)}$	$\frac{z[z(b-a) - (be^{-aT} - ae^{-bT})]}{(z-e^{-aT})(z-e^{-bT})}$	—
11. $\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$	$\frac{z \sin \omega T}{z^2 - 2z \cos \omega T + 1}$	$\sin \omega kT$
12. $\cos \omega t$	$\frac{s}{s^2 + \omega^2}$	$\frac{z(z - \cos \omega T)}{z^2 - 2z \cos \omega T + 1}$	$\cos \omega kT$
13. $e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$	$\frac{(ze^{-aT} \sin \omega T)}{z^2 - 2ze^{-aT} \cos \omega T + e^{-2aT}}$	$e^{-akT} \sin \omega kT$

APPENDIX 1

Range and signal overlapping	Process Convolution (Calculation)	End Value
Range: 		
Range: 		
Range: 		
Range: 		
Range:		

		
Range: 		
Range: 		
Range: 		
Range: 