CONFIDENTIAL

# UNIVERSITI KUALA LUMPUR
## MALAYSIAN INSTITUTE OF MARINE ENGINEERING TECHNOLOGY

## FINAL EXAMINATION
### SEPTEMBER 2016 SEMESTER

| | | |
|---|---|---|
| COURSE CODE | : | LGB12303 |
| COURSE NAME | : | MATHEMATICS 2 |
| PROGRAMME NAME (FOR MPU: PROGRAMME LEVEL) | : | BACHELOR ENGINEERING TECHNOLOGY OF MARINE ELECTRICAL AND ELECTRONIC |
| DATE | : | 19 JANUARY 2017 |
| TIME | : | 09.00 AM – 12.00 PM |
| DURATION | : | 3 HOURS |

## INSTRUCTIONS TO CANDIDATES

1. Please CAREFULLY read the instructions given in the question paper.
2. This question paper has information printed on both sides of the paper.
3. This question paper consists of TWO (2) sections; Section A and Section B.
4. Answer ALL questions in Section A. For Section B, answer THREE (3) questions.
5. Please write your answers on the answer booklet provided.
6. Answer all questions in English language ONLY

THERE ARE 6 PAGES OF QUESTIONS, INCLUDING THIS PAGE.

**PART A (Total: 40 marks)**

**INSTRUCTION: Answer ALL questions.**
**Please use the answer booklet provided.**

**Question 1**

(a) Calculate the expectation of obtaining a 4 upwards with 3 throws of fair dice.

(2 marks)

(b) Determine the probabilities of selecting at random the following options from a crowd containing 20 men and 33 women.

i. A man

(3 marks)

ii. A woman

(3 marks)

**Question 2**

(a) Determine the sixteenth term of the series 2, 7, 12, ... .

(3 marks)

(b) The $7^{th}$ term is twice than $5^{th}$ term. The third term of an arithmetic sequence is less than the fourth term by three. Determine the common difference and the first term.

(5 marks)

**Question 3**

If $p = 2i + j - k$ and $q = i - 3j + 2k$, determine

(a) $p + q$

(2 marks)

(b) $p \bullet q$, and

(3 marks)

(c) $|p| + |q|$

(3 marks)

**Question 4**

Solve $4xy\dfrac{dy}{dx} = y^2 - 1$

(8 marks)

**Question 5**

(a) By using the definition of Laplace transform, determine Laplace transform of $f(t) = 2$.

(8 marks)

**PART B (Total: 60 marks)**

**INSTRUCTION: Answer THREE questions.**
**Please use the answer booklet provided.**

**Question 6**

(a) Calculate the number of triangles that can be formed by joining the vertices of an octagon.

(3 marks)

(b) Discover the number of ways the letters of the word "KNOWLEDGE" can be arranged so that the vowels always come together.

(6 marks)

(c) A batch of 40 components contains 5 units which are defective. If a component is drawn at random from the batch and tested and then a second component is drawn at random, calculate the probability of

i. having one defective component both with replacement, and

(5 marks)

ii. having one defective component both without replacement.

(6 marks)

**Question 7**

(a) The first term of a geometric sequences is 12 and the fifth term is 55. Determine the eighth term of the sequence.

(3 marks)

(b) Calculate four terms that can be inserted between 5 and 22.5 to form an arithmetic sequences.

(5 marks)

(c) Determine which term of the sequence 2187, 729, 243, ... is $\frac{1}{9}$.

(7 marks)

(d) Determine the minimum number of terms in the sequence 3, 12, 48, 192, ...,196608 .

(5 marks)

**Question 8**

(a) Calculate the vector ***a*** joining point P and Q where point P has co-ordinates $(4, -1, 3)$ and point Q has co-ordinates $(2, 5, 0)$

(3 marks)

(b) Evaluate the moment and the magnitude of the moment of a force of $(i+2j-3k)$ newtons about point B having co-ordinates $(0, 1, 1)$, when the force acts in the line through A whose co-ordinates are $(1, 3, 4)$

(9 marks)

(c) The axis of a circular cylinder coincides with the $z$-axis and it rotates with an angular velocity of $(2i-5j+7k)$ rad/s. Determine the tangential velocity at a point P on the cylinder, whose co-ordinates are $(j+3k)$ meters and also determine the magnitude of the tangential velocity.

(8 marks)

**Question 9**

(a) Determine the general solution of $x\frac{dy}{dx} = 2 - 4x^3$

(3 marks)

(b) Solve the equation $2t\left(t - \frac{d\theta}{dt}\right) = 5$, given $\theta = 2$ when $t = 1$.

(7 marks)

(c) The equation of motion of a body oscillating at the end of a spring is

$$\frac{d^2x}{dt^2} + 100x = 0$$

where $x$ is the displacement in meters of the body from its equilibrium position after time $t$ seconds. Determine $x$ in terms of $t$ given that at time $t = 0, x = 2m$ and $\frac{dx}{dt} = 0$.

(10 marks)

**Question 10**

(a) Using a standard list of Laplace transforms to determine $L\left\{1 + 2t - \frac{1}{3}t^4\right\}$

(3 marks)

(b) Determine $L^{-1}\left\{\frac{4s-5}{s^2 - s - 2}\right\}$.

(10 marks)

(c) Use the Laplace transform of the second derivative to derive $L\{\cos at\} = \frac{s^2}{s^2 + a^2}$.

(7 marks)

**END OF QUESTION**

# DIFFERENTIATION

| STANDARD FORM | GENERAL FORM |
|---|---|
| $\frac{d}{dx}(\sin x)=\cos x$ | $\frac{d}{dx}(\sin f(x))=f'(x)\cos f(x)$ |
| $\frac{d}{dx}(\cos x)=-\sin x$ | $\frac{d}{dx}(\cos f(x))=-f'(x)\sin f(x)$ |
| $\frac{d}{dx}(\tan x)=\sec^2 x$ | $\frac{d}{dx}(\tan f(x))=f'(x)\sec^2 f(x)$ |
| $\frac{d}{dx}(\csc x)=-\csc x\cot x$ | $\frac{d}{dx}(\csc f(x))=-f'(x)\csc f(x)\cot f(x)$ |
| $\frac{d}{dx}(\sec x)=\sec x\tan x$ | $\frac{d}{dx}(\sec f(x))=f'(x)\sec f(x)\tan f(x)$ |
| $\frac{d}{dx}(\cot x)=-\csc^2 x$ | $\frac{d}{dx}(\cot f(x))=-f'(x)\csc^2 f(x)$ |

## EXPONENTIAL FUNCTION

| STANDARD FORM | GENERAL FORM |
|---|---|
| $\frac{d}{dx}e^x=e^x$ | $\frac{d}{dx}e^{f(x)}=f'(x)e^{f(x)}$ |

## LOGARITHMIC FUNCTION

| STANDARD FORM | GENERAL FORM |
|---|---|
| $\frac{d}{dx}\ln x=\frac{1}{x}$ | $\frac{d}{dx}\ln f(x)=\frac{f'(x)}{f(x)}$ |

## INTEGRATION

| STANDARD FORM | GENERAL FORM<br>Where : $f(x)=ax+b$ |
|---|---|
| $\int \cos x\,dx = \sin x + c$ | $\int \cos f(x)dx = \frac{\sin f(x)}{f'(x)} + c$ |
| $\int \sin x\,dx = -\cos x + c$ | $\int \sin f(x)dx = \frac{-\cos f(x)}{f'(x)} + c$ |
| $\int \sec^2 x\,dx = \tan x + c$ | $\int \sec^2 f(x)dx = \frac{\tan f(x)}{f'(x)} + c$ |
| $\int \sec x \tan x\,dx = \sec x + c$ | $\int \sec f(x) \tan f(x)dx = \frac{\sec f(x)}{f'(x)} + c$ |
| $\int \csc x \cot x\,dx = -\csc x + c$ | $\int \csc f(x) \cot f(x)dx = \frac{-\csc f(x)}{f'(x)} + c$ |
| $\int \csc^2 x\,dx = -\cot x + c$ | $\int \csc^2 f(x)dx = \frac{-\cot f(x)}{f'(x)} + c$ |
| $\int \tan x\,dx = \ln\lvert\sec x\rvert + c$ | $\int \tan x\,dx = \frac{\ln\lvert\sec f(x)\rvert}{f'(x)} + c$ |
| $\int \sec x\,dx = \ln\lvert\sec x + \tan x\rvert + c$ | $\int \sec x\ dx = \frac{\ln\lvert\sec f(x) + \tan f(x)\rvert}{f'(x)} + c$ |
| $\int \cot x\,dx = \ln\lvert\sin x\rvert + c$ | $\int \cot x\,dx = \frac{\ln\lvert\sin f(x)\rvert}{f'(x)} + c$ |
| $\int \csc x\,dx = -\ln\lvert\csc x + \cot x\rvert + c$ | $\int \csc x dx = \frac{-\ln\lvert\csc f(x) + \cot f(x)\rvert}{f'(x)} + c$ |

## EXPONENTIAL FUNCTION

| STANDARD FORM | GENERAL FORM<br>Where : $f(x)=ax+b$ |
|---|---|
| $\int e^x dx = e^x + c$ | $\int e^{f(x)}\,dx = \frac{e^{f(x)}}{f'(x)} + c$ |

## LOGARITHMIC FUNCTION

| STANDARD FORM | GENERAL FORM<br>Where : $f(x)=ax+b$ |
|---|---|
| $\int \frac{1}{x} dx = \ln\lvert x\rvert + c$ | $\int \frac{1}{f(x)} dx = \frac{\ln\lvert f(x)\rvert}{f'(x)} + c$ |

## LAPLACE TRANSFORM

| $Laplace\ transforms - Table$ | | | |
|---|---|---|---|
| $f(t) = L^{-1}\{F(s)\}$ | $F(s)$ | $f(t) = L^{-1}\{F(s)\}$ | $F(s)$ |
| $a \quad t \geq 0$ | $\frac{a}{s} \qquad s > 0$ | $\sin \omega t$ | $\frac{\omega}{s^2 + \omega^2}$ |
| $at \quad t \geq 0$ | $\frac{a}{s^2}$ | $\cos \omega t$ | $\frac{s}{s^2 + \omega^2}$ |
| $e^{-at}$ | $\frac{1}{s + a}$ | $\sin(\omega t + \theta)$ | $\frac{s \sin\theta + \omega \cos\theta}{s^2 + \omega^2}$ |
| $te^{-at}$ | $\frac{1}{(s + a)^2}$ | $\cos(\omega t + \theta)$ | $\frac{s \cos\theta - \omega \sin\theta}{s^2 + \omega^2}$ |
| $\frac{1}{2}t^2 e^{-at}$ | $\frac{1}{(s + a)^3}$ | $t \sin \omega t$ | $\frac{2\omega s}{(s^2 + \omega^2)^2}$ |
| $\frac{1}{(n-1)!}t^{n-1}e^{-at}$ | $\frac{1}{(s + a)^n}$ | $t \cos \omega t$ | $\frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$ |
| $e^{at}$ | $\frac{1}{s - a} \qquad s > a$ | $\sinh \omega t$ | $\frac{\omega}{s^2 - \omega^2} \qquad s > \lvert\omega\rvert$ |
| $te^{at}$ | $\frac{1}{(s - a)^2}$ | $\cosh \omega t$ | $\frac{s}{s^2 - \omega^2} \qquad s > \lvert\omega\rvert$ |
| $\frac{1}{b - a}(e^{-at} - e^{-bt})$ | $\frac{1}{(s + a)(s + b)}$ | $e^{-at} \sin \omega t$ | $\frac{\omega}{(s + a)^2 + \omega^2}$ |
| $\frac{1}{a^2}[1 - e^{-at}(1 + at)]$ | $\frac{1}{s(s + a)^2}$ | $e^{-at} \cos \omega t$ | $\frac{s + a}{(s + a)^2 + \omega^2}$ |
| $t^n$ | $\frac{n!}{s^{n+1}} \qquad n = 1,2,3..$ | $e^{at} \sin \omega t$ | $\frac{\omega}{(s - a)^2 + \omega^2}$ |
| $t^n e^{at}$ | $\frac{n!}{(s - a)^{n+1}} \qquad s > a$ | $e^{at} \cos \omega t$ | $\frac{s - a}{(s - a)^2 + \omega^2}$ |
| $t^n e^{-at}$ | $\frac{n!}{(s + a)^{n+1}} \qquad s > a$ | $1 - e^{-at}$ | $\frac{a}{s(s + a)}$ |
| $\sqrt{t}$ | $\frac{\sqrt{\pi}}{2s^{3/2}}$ | $\frac{1}{a^2}(at - 1 + e^{-at})$ | $\frac{1}{s^2(s + a)}$ |
| $\frac{1}{\sqrt{t}}$ | $\sqrt{\frac{\pi}{s}} \qquad s > 0$ | $f(t - t_1)$ | $e^{-t_1 s}F(s)$ |
| $g(t) \cdot p(t)$ | $G(s) \cdot P(s)$ | $f_1(t) \pm f_2(t)$ | $F_1(s) \pm F_2(s)$ |
| $\int f(t)dt$ | $\frac{F(s)}{s} + \frac{f^{-1}(0)}{s}$ | $\delta(t)$ *unit impulse* | $1 \qquad$ *all s* |
| $\frac{df}{dt}$ | $sF(s) - f(0)$ | $\frac{d^2 f}{df^2}$ | $s^2F(s) - sf(0) - f'(0)$ |
| $\frac{d^n f}{dt^n}$ | $s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - s^{n-3}f''(0) - \cdots - f^{n-1}(0)$ | | |

## FIRST AND SECOND ORDER DIFFERENTIAL EQUATION

If the roots of the auxiliary equation are:

(i) **real and different**, say $m = \alpha$ and $m = \beta$, then the general solution is

$$y = Ae^{\alpha x} + Be^{\beta x}$$

(ii) **real and equal, say** $m = \alpha$ twice, then the general solution is

$$y = (Ax + B)e^{\alpha x}$$

(iii) **complex**, say $m = \alpha \pm j\beta$, then the general solution is

$$y = e^{\alpha x}\{A \cos \beta x + B \sin \beta x\}$$

Table 51.1 Form of particular integral for different functions

| *Type* | *Straightforward cases* Try as particular integral: | *'Snag' cases* Try as particular integral: | *See problem* |
|---|---|---|---|
| (a) $f(x)=$ a constant | $v=k$ | $v=kx$ (used when C.F. contains a constant) | 1, 2 |
| (b) $f(x)=$ polynomial (i.e. $f(x)=L+Mx+Nx^2+\cdots$ where any of the coefficients may be zero) | $v=a+bx+cx^2+\cdots$ | | 3 |
| (c) $f(x)=$ an exponential function (i.e. $f(x)=Ae^{ax}$) | $v=ke^{ax}$ | (i) $v=kxe^{ax}$ (used when $e^{ax}$ appears in the C.F.) | 4, 5 |
| | | (ii) $v=kx^2e^{ax}$ (used when $e^{ax}$ **and** $xe^{ax}$ both appear in the C.F.) | 6 |
| (d) $f(x)=$ a sine or cosine function (i.e. $f(x)=a\sin px+b\cos px$, where $a$ or $b$ may be zero) | $v=A\sin px+B\cos px$ | $v=x(A\sin px+B\cos px)$ (used when $\sin px$ and/or $\cos px$ appears in the C.F.) | 7, 8 |
| (e) $f(x)=$ a sum e.g. | | | 9 |
| (i) $f(x)=4x^2-3\sin 2x$ | (i) $v=ax^2+bx+c$ $+d\sin 2x+e\cos 2x$ | | |
| (ii) $f(x)=2-x+e^{3x}$ | (ii) $v=ax+b+ce^{3x}$ | | |
| (f) $f(x)=$ a product e.g. $f(x)=2e^x\cos 2x$ | $v=e^x(A\sin 2x+B\cos 2x)$ | | 10 |