CONFIDENTIAL

# UNIVERSITI KUALA LUMPUR
## MALAYSIAN INSTITUTE OF MARINE ENGINEERING TECHNOLOGY

## FINAL EXAMINATION
### SEPTEMBER 2016 SEMESTER

| | | |
|---|---|---|
| COURSE CODE | : | LEB30503 |
| COURSE NAME | : | SIGNALS AND SYSTEMS |
| PROGRAMME NAME<br>(FOR MPU: PROGRAMME LEVEL) | : | BACHELOR OF ENGINEERING TECHNOLOGY IN MARINE ELECTRICAL AND ELECTRONICS |
| DATE | : | 25TH JANUARY 2017 |
| TIME | : | 09.00 AM – 12.00 PM |
| DURATION | : | 3 HOURS |

## INSTRUCTIONS TO CANDIDATES

1. Please CAREFULLY read the instructions given in the question paper.
2. This question paper has information printed on both sides of the paper.
3. This question paper consists of TWO (2) sections; Section A and Section B.
4. Answer ALL questions in Section A. For Section B, answer TWO (2) questions.
5. Please write your answers on the answer booklet provided.
6. Answer all questions in English language ONLY.

THERE ARE 6 PAGES OF QUESTIONS, EXCLUDING THIS PAGE.

## SECTION A (Total: 40 marks)

**INSTRUCTION: Answer ALL questions.**

**Please use the answer booklet provided.**

### Question 1

(a) Explain the principle concept of spectrum in term of signal processing. You are required to give one (1) related example for this explanation.

(12 marks)

(b) Explain basic power spectral density measurement using block diagram.

(8 marks)

*(Course Learning Outcome: 1)*

### Question 2

(a) Consider the continuous-time system defined by:

$$y(t) = \sin[x(t)]$$

Identify whether this system is time-invariant or time-varying system.

[5 marks]

(b) Distinguish between:

i. Continuous-time signals and discrete time signal

[5 marks]

ii. Even and odd signals.

[5 marks]

iii. Periodic and non-periodic signals

[5 marks]

*(Course Learning Outcome: 1)*

**SECTION B (Total: 60 marks)**

**INSTRUCTION: Answer THREE (3) questions ONLY.**
**Please use the answer booklet provided.**

**Question 3**

(a) Consider a continuous-time LTI system which has impulse response of $h(t) = \{u(t) - u(t-1)\}$. If $x(t) = t^2\{u(t) - u(t-3)\}$ is applied at the input of the system, evaluate the output $y(t)$ of the system using convolution integral $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\delta\tau$ as follows:

i. Sketch $x(\tau)$ and $h(t-\tau)$ for different intervals of '$t$'.

[6 marks]

ii. Evaluate the output $y(t)$ for the intervals of '$t$' indicated in Question 3(a)ii.

[5 marks]

*(Course Learning Outcome: 1)*

(b) Refer Figure 1(a), determine $y(t)$ using definition of Fourier Transform and sketch $Y(\omega)$ if $X(\omega)$ is given by Figure 1(b). Assume $\omega_c > \omega_0$.

*(Course Learning Outcome: 3)*

(9 marks)

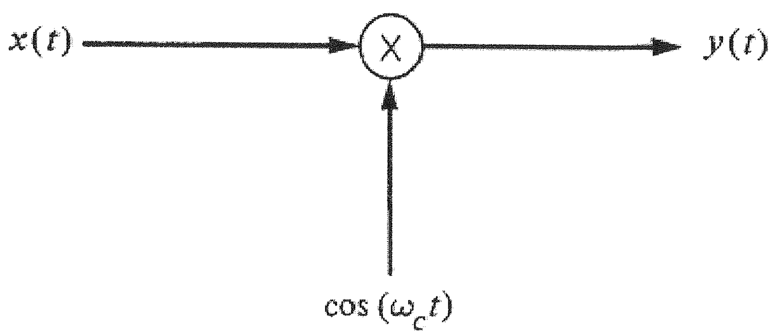


Figure 1(a)

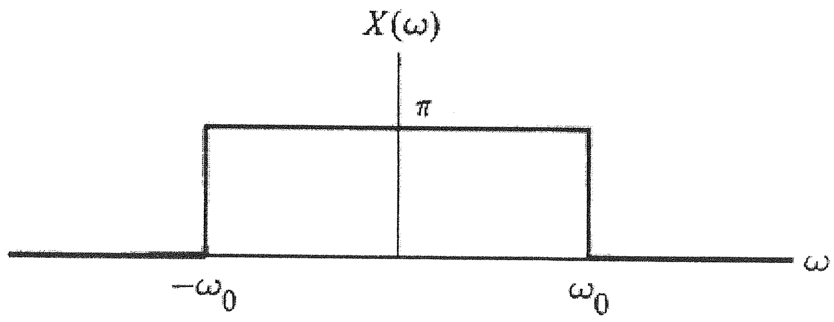


Figure 1(b)

**Question 4**

(a) Find the signal corresponding to the Fourier Transform shown in Figure 2.
*(Course Learning Outcome: 3)*

(10 marks)

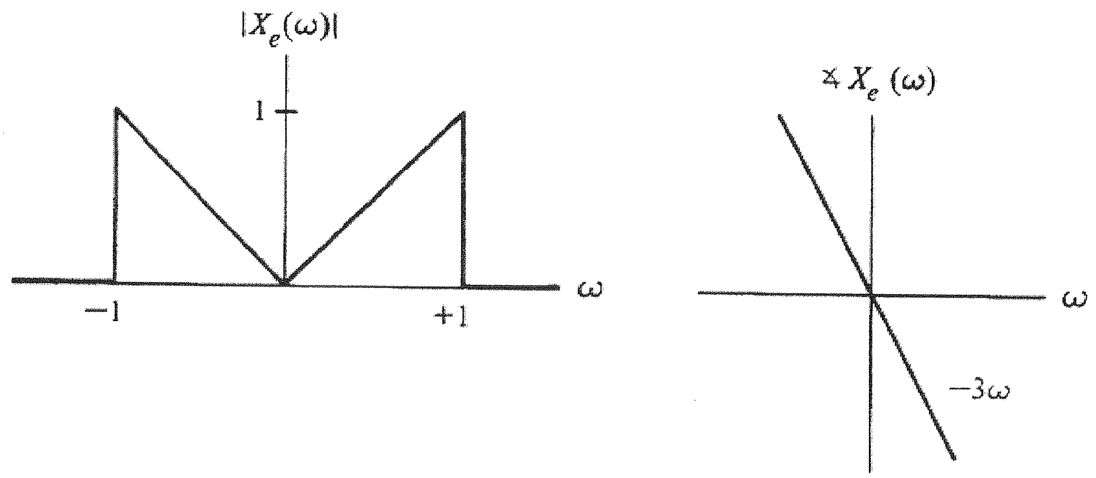


Figure 2

(b) For the filter specifications:

- Passband gain between 1 and 0.794 (-1dB) for $0 < \omega < 10$.
- Stopband gain which does not exceed 0.01 (-20dB) for $\omega > 30$.

Design a Butterworth filter by answering the following questions:

i. State the passband frequency $(\omega_p)$ and the stopband frequency $(\omega_s)$.

(1 mark)

ii. Sketch the filter specification.

(1 mark)

iii. Determine the filter order.

(2 mark)

iv. Determine the cut off frequency.

(1 mark)

v. Determine the Butterworth filter transfer function based on filter order determine in Question(b)iii.

(1 mark)

vi. Determine the normalized filter transfer function.

(1 mark)

vii. Identify how the filter order related to the transition band $(\omega_s - \omega_c)$.

(1 mark)

viii. Identify how the filter order related to the passband/stopband gain.

(2 marks)

The gain in dB follows as

$$\hat{G}_x = 10\log\left[H(\omega)H(\omega)^*\right] = -10\log\left[1+\left(\frac{\omega_x}{\omega_c}\right)^{2n}\right]$$

Where $x$ is either $p$ for passband or $s$ for stopband. The filter order is evaluated by

$$n = \frac{\log\left(\dfrac{10^{-\hat{G}_s/10}-1}{10^{-\hat{G}_p/10}-1}\right)}{2\log\left(\dfrac{\omega_s}{\omega_p}\right)}$$

*(Course Learning Outcome: 4)*

**Question 5**

(a) Determine the zero-state response of the system given by:

$$\frac{d^2 y(t)}{dt^2} + 3\frac{dy}{dt} + 2y(t) = 2x(t)$$

Given $y(0) = -1$, $\dot{y}(0) = 1$ and $x(t) = \cos(t)u(t)$.

*(Course Learning Outcome: 4)*

(12 marks)

(b) A system has impulse response $h[n] = \left(\frac{1}{2}\right)^n u[n]$, determine the input to the system if the output is given by $y[n] = \frac{1}{3}u[n] + \frac{2}{3}\left(-\frac{1}{2}\right)^n u[n]$ using z-transform method.

*(Course Learning Outcome: 3)*

(8 marks)

**Question 6**

(a) Refer to Figure 3, determine a differential equation that relates $v_i$ to $v_c$ as follows. First, determine relations among the element currents ($i_R, i_L$, and $i_c$) and element voltages ($v_R, v_L$, and $v_c$) using KVL and KCL. Second, relate each element voltage to the corresponding element current using the constitutive relation for the element: i.e. $v_R = ir_R$, $i_c = C\frac{dv_c}{dt}$, and $v_L = L\frac{di_L}{dt}$. Finally, solve your equations to find a single equation with terms that involve $v_i$, $v_o$, and derivatives of $v_i$ and $v_o$.

*(Course Learning Outcome: 3)*

(4 marks)

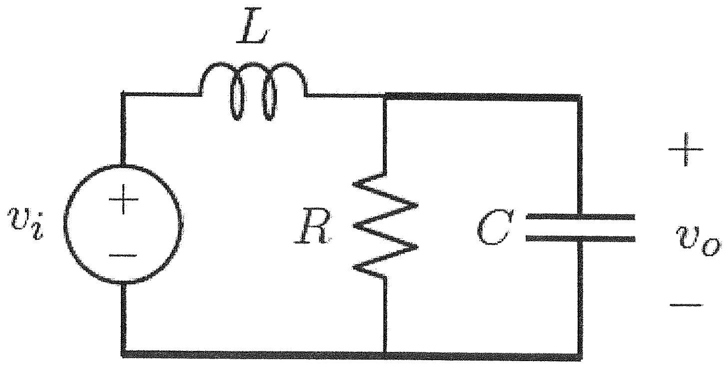


Figure 3

(b) Determine the system function $H(s) = \frac{V_0(s)}{V_i(s)}$, based on the Laplace transform of your answer from Question 6(a).

(5 marks)

(c) An FM (Frequency Modulation) transmission system has the following characteristics:

- Transmission frequency: $f_i = 100Hz$ and $\omega_i = 2\pi f_i$.
- Maximum frequency of the signal to be transmitted: $f_{\max} = 10Hz$.
- Voltage-controlled oscillator (VCO) gain: $K_o = 1$
- Loop filter: third order Butterworth.

Answer the following questions:

i. Determine the cutoff frequency of the loop filter. Discuss the reasons behind the selected frequency.

ii. Evaluate the transfer function of the loop filter

*(Course Learning Outcome: 4)*

(11 marks)

**END OF EXAMINATION PAPER**

## Table of formulae for LEB30503 Signals and Systems

*(For use during examination only)*

Convolution Table

| No. | $x_1(t)$ | $x_2(t)$ | $x_1(t) * x_2(t) = x_2(t) * x_1(t)$ |
|---|---|---|---|
| 1 | $x(t)$ | $\delta(t-T)$ | $x(t-T)$ |
| 2 | $e^{\lambda t}u(t)$ | $u(t)$ | $\frac{1-e^{\lambda t}}{-\lambda}u(t)$ |
| 3 | $u(t)$ | $u(t)$ | $tu(t)$ |
| 4 | $e^{\lambda_1 t}u(t)$ | $e^{\lambda_2 t}u(t)$ | $\frac{e^{\lambda_1 t}-e^{\lambda_2 t}}{\lambda_1-\lambda_2}u(t) \quad \lambda_1 \neq \lambda_2$ |
| 5 | $e^{\lambda t}u(t)$ | $e^{\lambda t}u(t)$ | $te^{\lambda t}u(t)$ |
| 6 | $te^{\lambda t}u(t)$ | $e^{\lambda t}u(t)$ | $\frac{1}{2}t^2e^{\lambda t}u(t)$ |
| 7 | $t^N u(t)$ | $e^{\lambda t}u(t)$ | $\frac{N!\,e^{\lambda t}}{\lambda^{N+1}}u(t) - \sum_{k=0}^{N}\frac{N!\,t^{N-k}}{\lambda^{k+1}(N-k)!}u(t)$ |
| 8 | $t^M u(t)$ | $t^N u(t)$ | $\frac{M!N!}{(M+N+1)!}t^{M+N+1}u(t)$ |
| 9 | $te^{\lambda_1 t}u(t)$ | $e^{\lambda_2 t}u(t)$ | $\frac{e^{\lambda_2 t}-e^{\lambda_1 t}+(\lambda_1-\lambda_2)te^{\lambda_1 t}}{(\lambda_1-\lambda_2)^2}u(t)$ |
| 10 | $t^M e^{\lambda t}u(t)$ | $t^N e^{\lambda t}u(t)$ | $\frac{M!\,N!}{(N+M+1)!}t^{M+N+1}e^{\lambda t}u(t)$ |
| 11 | $t^M e^{\lambda_1 t}u(t)$ <br> $\lambda_1 \neq \lambda_2$ | $t^N e^{\lambda_2 t}u(t)$ | $\sum_{k=0}^{M}\frac{(-1)^k M!(N+k)!\,t^{M-k}e^{\lambda_1 t}}{k!(M-k)!(\lambda_1-\lambda_2)^{N+k+1}}u(t) + \sum_{k=0}^{N}\frac{(-1)^k N!(M+k)!\,t^{N-k}e^{\lambda_2 t}}{k!(N-k)!(\lambda_2-\lambda_1)^{M+k+1}}u(t)$ |
| 12 | $e^{-\alpha t}\cos(\beta t+\theta)u(t)$ | $e^{\lambda t}u(t)$ | $\frac{\cos(\theta-\phi)e^{\lambda t}-e^{-\alpha t}\cos(\beta t+\theta-\phi)}{\sqrt{(\alpha+\lambda)^2+\beta^2}}u(t)$ <br> $\phi = \tan^{-1}[-\beta/(\alpha+\lambda)]$ |
| 13 | $e^{\lambda_1 t}u(t)$ | $e^{\lambda_2 t}u(-t)$ | $\frac{e^{\lambda_1 t}u(t)+e^{\lambda_2 t}u(-t)}{\lambda_2-\lambda_1} \quad \mathrm{Re}\,\lambda_2 > \mathrm{Re}\,\lambda_1$ |
| 14 | $e^{\lambda_1 t}u(-t)$ | $e^{\lambda_2 t}u(-t)$ | $\frac{e^{\lambda_1 t}-e^{\lambda_2 t}}{\lambda_2-\lambda_1}u(-t)$ |

**Table of formulae for LEB30503 Signals and Systems**

*(For use during examination only)*

Laplace Transform Table

| No. | $x(t)$ | $X(s)$ |
|---|---|---|
| 1 | $\delta(t)$ | $1$ |
| 2 | $u(t)$ | $\frac{1}{s}$ |
| 3 | $tu(t)$ | $\frac{1}{s^2}$ |
| 4 | $t^n u(t)$ | $\frac{n!}{s^{n+1}}$ |
| 5 | $e^{\lambda t} u(t)$ | $\frac{1}{s-\lambda}$ |
| 6 | $te^{\lambda t} u(t)$ | $\frac{1}{(s-\lambda)^2}$ |
| 7 | $t^n e^{\lambda t} u(t)$ | $\frac{n!}{(s-\lambda)^{n+1}}$ |
| 8a | $\cos bt\, u(t)$ | $\frac{s}{s^2+b^2}$ |
| 8b | $\sin bt\, u(t)$ | $\frac{b}{s^2+b^2}$ |
| 9a | $e^{-at}\cos bt\, u(t)$ | $\frac{s+a}{(s+a)^2+b^2}$ |
| 9b | $e^{-at}\sin bt\, u(t)$ | $\frac{b}{(s+a)^2+b^2}$ |
| 10a | $re^{-at}\cos(bt+\theta)\, u(t)$ | $\frac{(r\cos\theta)s+(ar\cos\theta-br\sin\theta)}{s^2+2as+(a^2+b^2)}$ |
| 10b | $re^{-at}\cos(bt+\theta)\, u(t)$ | $\frac{0.5re^{j\theta}}{s+a-jb}+\frac{0.5re^{-j\theta}}{s+a+jb}$ |
| 10c | $re^{-at}\cos(bt+\theta)\, u(t)$; $r=\sqrt{\frac{A^2c+B^2-2ABa}{c-a^2}}$; $\theta=\tan^{-1}\left(\frac{Aa-B}{A\sqrt{c-a^2}}\right)$; $b=\sqrt{c-a^2}$ | $\frac{As+B}{s^2+2as+c}$ |
| 10d | $e^{-at}\left[A\cos bt+\frac{B-Aa}{b}\sin bt\right]u(t)$; $b=\sqrt{c-a^2}$ | $\frac{As+B}{s^2+2as+c}$ |

## Table of formulae for LEB30503 Signals and Systems

*(For use during examination only)*

Summary of Laplace Transform Operation

| Operation | $x(t)$ | $X(s)$ |
|---|---|---|
| Addition | $x_1(t) + x_2(t)$ | $X_1(s) + X_2(s)$ |
| Scalar multiplication | $kx(t)$ | $kX(s)$ |
| Time differentiation | $\frac{dx}{dt}$ | $sX(s) - x(0^-)$ |
| | $\frac{d^2x}{dt^2}$ | $s^2X(s) - sx(0^-) - \dot{x}(0^-)$ |
| | $\frac{d^3x}{dt^3}$ | $s^3X(s) - s^2x(0^-) - s\dot{x}(0^-) - \ddot{x}(0^-)$ |
| | $\frac{d^nx}{dt^n}$ | $s^nX(s) - \sum_{k=1}^{n} s^{n-k}x^{(k-1)}(0^-)$ |
| Time integration | $\int_{0^-}^{t} x(\tau)\,d\tau$ | $\frac{1}{s}X(s)$ |
| | $\int_{-\infty}^{t} x(\tau)\,d\tau$ | $\frac{1}{s}X(s) + \frac{1}{s}\int_{-\infty}^{0^-} x(t)\,dt$ |

| Operation | $x(t)$ | $X(s)$ | |
|---|---|---|---|
| Time shifting | $x(t - t_0)u(t - t_0)$ | $X(s)e^{-st_0}$ | $t_0 \geq 0$ |
| Frequency shifting | $x(t)e^{s_0t}$ | $X(s - s_0)$ | |
| Frequency differentiation | $-tx(t)$ | $\frac{dX(s)}{ds}$ | |
| Frequency integration | $\frac{x(t)}{t}$ | $\int_{s}^{\infty} X(z)\,dz$ | |
| Scaling | $x(at), a \geq 0$ | $\frac{1}{a}X\left(\frac{s}{a}\right)$ | |
| Time convolution | $x_1(t) * x_2(t)$ | $X_1(s)X_2(s)$ | |
| Frequency convolution | $x_1(t)x_2(t)$ | $\frac{1}{2\pi j}X_1(s) * X_2(s)$ | |
| Initial value | $x(0^+)$ | $\lim_{s\to\infty} sX(s)$ | $(n > m)$ |
| Final value | $x(\infty)$ | $\lim_{s\to 0} sX(s)$ | [poles of $sX(s)$ in LHP] |

## Table of formulae for LEB30503 Signals and Systems

*(For use during examination only)*

Fourier Transform Table

| No. | $x(t)$ | $X(\omega)$ | |
|---|---|---|---|
| 1 | $e^{-at}u(t)$ | $\dfrac{1}{a+j\omega}$ | $a>0$ |
| 2 | $e^{at}u(-t)$ | $\dfrac{1}{a-j\omega}$ | $a>0$ |
| 3 | $e^{-a\lvert t\rvert}$ | $\dfrac{2a}{a^2+\omega^2}$ | $a>0$ |
| 4 | $te^{-at}u(t)$ | $\dfrac{1}{(a+j\omega)^2}$ | $a>0$ |
| 5 | $t^n e^{-at}u(t)$ | $\dfrac{n!}{(a+j\omega)^{n+1}}$ | $a>0$ |
| 6 | $\delta(t)$ | $1$ | |
| 7 | $1$ | $2\pi\delta(\omega)$ | |
| 8 | $e^{j\omega_0 t}$ | $2\pi\delta(\omega-\omega_0)$ | |
| 9 | $\cos\omega_0 t$ | $\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$ | |
| 10 | $\sin\omega_0 t$ | $j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$ | |
| 11 | $u(t)$ | $\pi\delta(\omega)+\dfrac{1}{j\omega}$ | |
| 12 | $\operatorname{sgn} t$ | $\dfrac{2}{j\omega}$ | |
| 13 | $\cos\omega_0 t\,u(t)$ | $\dfrac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\dfrac{j\omega}{\omega_0^2-\omega^2}$ | |
| 14 | $\sin\omega_0 t\,u(t)$ | $\dfrac{\pi}{2j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]+\dfrac{\omega_0}{\omega_0^2-\omega^2}$ | |
| 15 | $e^{-at}\sin\omega_0 t\,u(t)$ | $\dfrac{\omega_0}{(a+j\omega)^2+\omega_0^2}$ | $a>0$ |
| 16 | $e^{-at}\cos\omega_0 t\,u(t)$ | $\dfrac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$ | $a>0$ |

| | | | |
|---|---|---|---|
| 16 | $e^{-at}\cos\omega_0 t\, u(t)$ | $\dfrac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$ | $a>0$ |
| 17 | $\text{rect}\left(\dfrac{t}{\tau}\right)$ | $\tau\,\text{sinc}\left(\dfrac{\omega\tau}{2}\right)$ | |
| 18 | $\dfrac{W}{\pi}\,\text{sinc}\,(Wt)$ | $\text{rect}\left(\dfrac{\omega}{2W}\right)$ | |
| 19 | $\Delta\left(\dfrac{t}{\tau}\right)$ | $\dfrac{\tau}{2}\,\text{sinc}^2\left(\dfrac{\omega\tau}{4}\right)$ | |
| 20 | $\dfrac{W}{2\pi}\,\text{sinc}^2\left(\dfrac{Wt}{2}\right)$ | $\Delta\left(\dfrac{\omega}{2W}\right)$ | |
| 21 | $\sum_{n=-\infty}^{\infty}\delta(t-nT)$ | $\omega_0\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0)$ | $\omega_0=\dfrac{2\pi}{T}$ |
| 22 | $e^{-t^2/2\sigma^2}$ | $\sigma\sqrt{2\pi}\,e^{-\sigma^2\omega^2/2}$ | |

**Table of formulae for LEB30503 Signals and Systems**

*(For use during examination only)*

Summary of Fourier Transform Operation

| Operation | $x(t)$ | $X(\omega)$ |
|---|---|---|
| Scalar multiplication | $kx(t)$ | $kX(\omega)$ |
| Addition | $x_1(t) + x_2(t)$ | $X_1(\omega) + X_2(\omega)$ |
| Conjugation | $x^*(t)$ | $X^*(-\omega)$ |
| Duality | $X(t)$ | $2\pi x(-\omega)$ |
| Scaling ($a$ real) | $x(at)$ | $\frac{1}{\lvert a\rvert} X\left(\frac{\omega}{a}\right)$ |
| Time shifting | $x(t - t_0)$ | $X(\omega)e^{-j\omega t_0}$ |
| Frequency shifting ($\omega_0$ real) | $x(t)e^{j\omega_0 t}$ | $X(\omega - \omega_0)$ |

| Operation | $x(t)$ | $X(\omega)$ |
|---|---|---|
| Time convolution | $x_1(t) * x_2(t)$ | $X_1(\omega)X_2(\omega)$ |
| Frequency convolution | $x_1(t)x_2(t)$ | $\frac{1}{2\pi} X_1(\omega) * X_2(\omega)$ |
| Time differentiation | $\frac{d^n x}{dt^n}$ | $(j\omega)^n X(\omega)$ |
| Time integration | $\int_{-\infty}^{t} x(u)\, du$ | $\frac{X(\omega)}{j\omega} + \pi X(0)\delta(\omega)$ |

**Table of formulae for LEB30503 Signals and Systems**
*(For use during examination only)*

Z- Transform Table

| | $f[k]$ | $F[z]$ |
|---|---|---|
| 1 | $\delta[k-j]$ | $z^{-j}$ |
| 2 | $u[k]$ | $\frac{z}{z-1}$ |
| 3 | $ku[k]$ | $\frac{z}{(z-1)^2}$ |
| 4 | $k^2u[k]$ | $\frac{z(z+1)}{(z-1)^3}$ |
| 5 | $k^3u[k]$ | $\frac{z(z^2+4z+1)}{(z-1)^4}$ |
| 6 | $\gamma^{k-1}u[k-1]$ | $\frac{1}{z-\gamma}$ |
| 7 | $\gamma^k u[k]$ | $\frac{z}{z-\gamma}$ |
| 8 | $k\gamma^k u[k]$ | $\frac{\gamma z}{(z-\gamma)^2}$ |
| 9 | $k^2\gamma^k u[k]$ | $\frac{\gamma z(z+\gamma)}{(z-\gamma)^3}$ |
| 10 | $\frac{k(k-1)(k-2)\cdots(k-m+1)}{\gamma^m m!}\gamma^k u[k]$ | $\frac{z}{(z-\gamma)^{m+1}}$ |

| | | |
|---|---|---|
| 11$a$ | $\lvert\gamma\rvert^k \cos \beta k\, u[k]$ | $\dfrac{z(z - \lvert\gamma\rvert \cos \beta)}{z^2 - (2\lvert\gamma\rvert \cos \beta) z + \lvert\gamma\rvert^2}$ |
| 11$b$ | $\lvert\gamma\rvert^k \sin \beta k\, u[k]$ | $\dfrac{z\lvert\gamma\rvert \sin \beta}{z^2 - (2\lvert\gamma\rvert \cos \beta) z + \lvert\gamma\rvert^2}$ |
| 12$a$ | $r\lvert\gamma\rvert^k \cos(\beta k + \theta) u[k]$ | $\dfrac{rz[z \cos \theta - \lvert\gamma\rvert \cos(\beta - \theta)]}{z^2 - (2\lvert\gamma\rvert \cos \beta) z + \lvert\gamma\rvert^2}$ |
| 12$b$ | $r\lvert\gamma\rvert^k \cos(\beta k + \theta) u[k] \qquad \gamma = \lvert\gamma\rvert e^{j\beta}$ | $\dfrac{(0.5re^{j\theta})z}{z - \gamma} + \dfrac{(0.5re^{-j\theta})z}{z - \gamma^*}$ |
| 12$c$ | $r\lvert\gamma\rvert^k \cos(\beta k + \theta) u[k]$ <br> $r = \sqrt{\frac{A^2\lvert\gamma\rvert^2 + B^2 - 2AaB}{\lvert\gamma\rvert^2 - a^2}}$ <br> $\beta = \cos^{-1} \frac{-a}{\lvert\gamma\rvert}, \ \theta = \tan^{-1} \frac{Aa - B}{A\sqrt{\lvert\gamma\rvert^2 - a^2}}$ | $\dfrac{z(Az + B)}{z^2 + 2az + \lvert\gamma\rvert^2}$ |

## Table of formulae for LEB30503 Signals and Systems

*(For use during examination only)*

Summary of Z-Transform Operation

| Operation | $f[k]$ | $F[z]$ |
|---|---|---|
| Addition | $f_1[k] + f_2[k]$ | $F_1[z] + F_2[z]$ |
| Scalar multiplication | $af[k]$ | $aF[z]$ |
| Right-shift | $f[k-m]u[k-m]$ | $\frac{1}{z^m}F[z]$ |
| | $f[k-m]u[k]$ | $\frac{1}{z^m}F[z] + \frac{1}{z^m}\sum_{k=1}^{m} f[-k]z^k$ |
| | $f[k-1]u[k]$ | $\frac{1}{z}F[z] + f[-1]$ |
| | $f[k-2]u[k]$ | $\frac{1}{z^2}F[z] + \frac{1}{z}f[-1] + f[-2]$ |
| | $f[k-3]u[k]$ | $\frac{1}{z^3}F[z] + \frac{1}{z^2}f[-1] + \frac{1}{z}f[-2] + f[-3]$ |
| Left-shift | $f[k+m]u[k]$ | $z^m F[z] - z^m \sum_{k=0}^{m-1} f[k]z^{-k}$ |
| | $f[k+1]u[k]$ | $zF[z] - zf[0]$ |
| | $f[k+2]u[k]$ | $z^2F[z] - z^2f[0] - zf[1]$ |
| | $f[k+3]u[k]$ | $z^3F[z] - z^3f[0] - z^2f[1] - zf[2]$ |
| Multiplication by $\gamma^k$ | $\gamma^k f[k]u[k]$ | $F\left[\frac{z}{\gamma}\right]$ |
| Multiplication by $k$ | $kf[k]u[k]$ | $-z\frac{d}{dz}F[z]$ |
| Time Convolution | $f_1[k] * f_2[k]$ | $F_1[z]F_2[z]$ |
| Frequency Convolution | $f_1[k]f_2[k]$ | $\frac{1}{2\pi j}\oint F_1[u]F_2\left[\frac{z}{u}\right]u^{-1}\,du$ |
| Initial value | $f[0]$ | $\lim_{z\to\infty} F[z]$ |
| Final value | $\lim_{N\to\infty} f[N]$ | $\lim_{z\to 1}(z-1)F[z]$ poles of $(z-1)F[z]$ inside the unit circle. |

## Table of formulae for LEB30503 Signals and Systems

*(For use during examination only)*

## Formulae

*Butterworth Filters (BF):*

Roots of the Butterworth polynomial,

$$s_m = -\sin[(2m-1)(\pi/2n)] + j\cos[(2m-1)(\pi/2n)] = \sigma_m + j\omega_m; \qquad m = 1,2,\ldots,2n$$

*List of polynomials Butterworth Filters (BF) up to n=7:*

| $n$ | *Polynomial* |
|---|---|
| 1 | $(s + 1)$ |
| 2 | $(s^2 + 1.414s + 1)$ |
| 3 | $(s + 1)(s^2 + s + 1)$ |
| 4 | $(s^2 + 0.7654s + 1)(s^2 + 1.8478s + 1)$ |
| 5 | $(s + 1)(s^2 + 0.6180s + 1)(s^2 + 1.6180s + 1)$ |
| 6 | $(s^2 + 0.5176s + 1)(s^2 + 1.4142s + 1)(s^2 + 1.939s + 1)$ |

*Chebyshev Filters (CF):*

Minimum value of ripple (dB),

$$dB(\gamma) = 10\log_{10}(1+\varepsilon^2)$$

Roots of the Chebyshev polynomial,

$$s_m = -\left[\sin\left((2m-1)\left(\frac{\pi}{2n}\right)\right)\right]\sinh\left[\left(\frac{1}{n}\right)\sinh^{-1}\left(\frac{1}{\varepsilon}\right)\right] + j\left[\cos\left((2m-1)\left(\frac{\pi}{2n}\right)\right)\right]\cosh\left[\left(\frac{1}{n}\right)\sinh^{-1}\left(\frac{1}{\varepsilon}\right)\right]$$

Transfer function of CF for normalized frequency,

## Table of formulae for LEB30503 Signals and Systems

*(For use during examination only)*

$$H_{c_n}(s) = \frac{K}{(-1)^n \prod_{m=1}^{n}\left(\frac{s}{s_m} - 1\right)}$$, K = specified gain.

Transfer function of CF for denormalized frequency,

$$H_{c_n}(s) = \frac{K}{(-1)^n \prod_{m=1}^{n}\left(\frac{s}{s_m \omega_c} - 1\right)}$$

*List of polynomials Chebyshev Filters (CF) up to n=8:*

| $n$ | $C_n(\omega)$ |
|---|---|
| 0 | 1 |
| 1 | $\omega$ |
| 2 | $2\omega^2 - 1$ |
| 3 | $4\omega^3 - 3\ \omega$ |
| 4 | $8\omega^4 - 8\omega^2 + 1$ |
| 5 | $16\omega^5 - 20\omega^3 + 5\omega$ |
| 6 | $32\omega^7 - 48\omega^4 + 18\omega^2 - 1$ |
| 7 | $74\omega^7 - 112\omega^5 + 57\omega^3 - 7\omega$ |
| 8 | $128\omega^8 - 256\omega^7 + 170\omega^4 - 32\omega^2_{+1}$ |

*FIR FILTER:*

$$H_{lc}(z) = H_l(z)\ z^{-(N-1)/2}$$

$$H_l(z) = h(0) + \sum_{\ell=1}^{\frac{N-1}{2}} h(\ell T)\,(z^{\ell} + z^{-\ell})$$