UNIVERSITI KUALA LUMPUR
MALAYSIAN INSTITUTE OF MARINE ENGINEERING TECHNOLOGY

FINAL EXAMINATION
JANUARY 2016 SEMESTER

COURSE CODE	:	LGB 22003
COURSE NAME	:	MATHEMATICS 3
PROGRAMME NAME	:	BACHELOR OF ENGINEERING TECHNOLOGY IN MARINE ELECTRICAL AND ELECTRONICS ENGINEERING
DATE	:	27 MAY 2016
TIME	:	09.00 AM – 12.00 PM
DURATION	:	3 HOURS

INSTRUCTIONS TO CANDIDATES

1. Please CAREFULLY read the instructions given in the question paper.
2. This question paper has information printed on both sides of the paper.
3. This question paper consists of TWO (2) sections; Section A and Section B.
4. Answer ALL FIVE (5) questions in Section A. For Section B, answer THREE (3) questions ONLY.
5. Please write your answers on answer sheet provided.
6. Answer all questions in English language ONLY.
7. FORMULA has been appended for your reference.

THERE ARE 6 PAGES OF QUESTIONS, INCLUDING THIS PAGE.

SECTION A (Total: 40 marks)

INSTRUCTION: Answer ALL FIVE questions.
Please use the answer booklet provided.

Question 1

(a) If given $g(z) = z + 2$ and $f(z) = z + 1$, show that the multiplication of $f(z)$ and $g(z)$ satisfies the Cauchy-Riemann which analytic in some region of complex variable.

(4 marks)

(b) Determine the residue of $f(z) = \dfrac{2z}{(z^2 - 1)}$ at each of its poles in the finite Z plane.

(4 marks)

Question 2

(a) Determine the second order partial derivatives of $f(x, y) = x^2 y^3 + x^4 y$ for:

i) $\dfrac{d^2 f}{dxdy}$

(2 marks)

ii) $\dfrac{d^2 f}{dydx}$

(2 marks)

(b) Determine the third order partial derivatives of $f(x, y) = y^2 e^x + y$ for f_{xyy} .

(4 marks)

Question 3

Given the unit impulse response $g(t) = 5e^{6t}$. Define the convolution between the unit impulse responses to

(a) a step input function

(4 marks)

(b) an exponential function, $f(t) = e^{-2t}$

(4 marks)

Question 4

By using the table of Fourier transform, solve Fourier transform by transforming the following function.

(a) $f(t) = 9\sin c\left(\frac{3(t-4)}{7}\right)$

(4 marks)

(b) $f(t) = 6rect(t)$ for $f(3t-6)$

(4 marks)

Question 5

Determine the z-transform of the following function.

(a) $\frac{a^n}{n!}e^{-a}$

(4 marks)

(b) $(n+1)^2$

(4 marks)

SECTION B (Total Marks: 100 marks)

INSTRUCTION: Answer only THREE questions (60 marks).
Please use the answer booklet provided.

Question 6

(a) Integrate of $f(z)=\dfrac{4z^3-20}{16z-i}$ using Cauchy Integral formula.

(4 marks)

(b) Given the contour integral $\oint_c \dfrac{3z^2+2}{(z-1)(z^2+9)}dz$ if C is:

i. Determine the residue for each poles in the finite plane.

(10 marks)

ii. Define Cauchy residue of the contour at circle $|z|=1.5$ and $|z|=15.5$

(6 marks)

Question 7

(a) Suppose that $w=x^2+y^2-z^2$ and $x=\rho\sin\phi\cos\theta,\ y=\rho\sin\phi\sin\theta,\ z=\rho\cos\phi$, use appropriate forms of the chain rule to determine:

i. Tree diagram and the formulas of partial derivatives flow.

(4 marks)

ii. $\dfrac{d\omega}{d\rho}$ and $\dfrac{d\omega}{d\theta}$

(10 marks)

(b) Let $t = xy$, so that $z = f(t)$. Show that when f is differentiable, a function of the form $z = f(x, y)$ satisfies the equation $x\frac{dz}{dx} - y\frac{dz}{dy} = 0$

(6 marks)

Question 8

Convolve the following two functions as shown in Figure 1.

(20 marks)

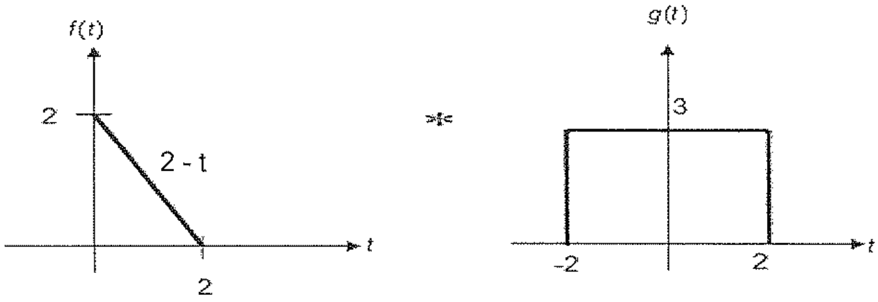

Figure 1

Question 9

(a) Determine Fourier Transform of convolution $f(t) = 6rect(5t)$ and $x'(t)$ if given $x(t) = 2\delta(t-9)$.

(10 marks)

(b) Define the definition of Fourier transform of $x(t) = t$ for the range of $0 \le t < \infty$.

(10 marks)

Question 10

Solve the inverse z-transform of the following function

(a) $X(z) = \dfrac{z}{2z^2 - 3z + 1}$ for $|z| < \dfrac{1}{2}$

(10 marks)

(b) $X(z) = \dfrac{2z^3 - 5z^2 + z + 3}{(z-1)(z-2)}$ for $|z| < 1$

(10 marks)

END OF QUESTIONS

TRIGONOMETRY IDENTITIES

FUNDAMENTAL IDENTITIES	FORMULAS FOR NEGATIVES
$\csc\theta = \frac{1}{\sin\theta}$	$\sin(-\theta) = -\sin\theta$
$\sec\theta = \frac{1}{\cos\theta}$	$\cos(-\theta) = \cos\theta$
$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$	$\tan(-\theta) = -\tan\theta$
$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$	$\csc(-\theta) = -\csc\theta$
$\sin^2\theta + \cos^2\theta = 1$	$\sec(-\theta) = \sec\theta$
$1+\tan^2\theta = \sec^2\theta$	$\cot(-\theta) = -\cot\theta$
$1+\cot^2\theta = \csc^2\theta$	$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$

ADDITION FORMULAS	SUBTRACTION FORMULAS
$\sin(A+B) = \sin A\cos B + \cos A\sin B$	$\sin(A-B) = \sin A\cos B - \cos A\sin B$
$\cos(A+B) = \cos A\cos B - \sin A\sin B$	$\cos(A-B) = \cos A\cos B + \sin A\sin B$
$\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A\tan B}$	$\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A\tan B}$

HALF-ANGLE FORMULAS	DOUBLE-ANGLE FORMULAS
$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$	$\sin 2\theta = 2\sin\theta\cos\theta$
$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$	$\cos 2\theta = \cos^2\theta - \sin^2\theta$ .... $= 1-2\sin^2\theta$ .... $= 2\cos^2\theta - 1$
$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$	$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

PRODUCT-TO-SUM FORMULAS	SUM-TO-PRODUCT FORMULAS
$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$	$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$
$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta)-\sin(\alpha-\beta)]$	$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$
$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)]$	$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

DIFFERENTIATION

STANDARD FORM	GENERAL FORM
$\frac{d}{dx}(\sin x) = \cos x$	$\frac{d}{dx}(\sin f(x)) = f'(x)\cos f(x)$
$\frac{d}{dx}(\cos x) = -\sin x$	$\frac{d}{dx}(\cos f(x)) = -f'(x)\sin f(x)$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\frac{d}{dx}(\tan f(x)) = f'(x)\sec^2 f(x)$
$\frac{d}{dx}(\csc x) = -\csc x\cot x$	$\frac{d}{dx}(\csc f(x)) = -f'(x)\csc f(x)\cot f(x)$
$\frac{d}{dx}(\sec x) = \sec x\tan x$	$\frac{d}{dx}(\sec f(x)) = f'(x)\sec f(x)\tan f(x)$
$\frac{d}{dx}(\cot x) = -\csc^2 x$	$\frac{d}{dx}(\cot f(x)) = -f'(x)\csc^2 f(x)$

EXPONENTIAL FUNCTION

STANDARD FORM	GENERAL FORM
$\frac{d}{dx}e^x = e^x$	$\frac{d}{dx}e^{f(x)} = f'(x)e^{f(x)}$

LOGARITHMIC FUNCTION

STANDARD FORM	GENERAL FORM
$\frac{d}{dx}\ln x = \frac{1}{x}$	$\frac{d}{dx}\ln f(x) = \frac{f'(x)}{f(x)}$

INTEGRATION

STANDARD FORM	GENERAL FORM Where : $f(x) = ax + b$
$\int \cos x \, dx = \sin x + c$	$\int \cos f(x) dx = \frac{\sin f(x)}{f'(x)} + c$
$\int \sin x \, dx = -\cos x + c$	$\int \sin f(x) dx = \frac{-\cos f(x)}{f'(x)} + c$
$\int \sec^2 x \, dx = \tan x + c$	$\int \sec^2 f(x) dx = \frac{\tan f(x)}{f'(x)} + c$
$\int \sec x \tan x \, dx = \sec x + c$	$\int \sec f(x) \tan f(x) dx = \frac{\sec f(x)}{f'(x)} + c$
$\int \csc x \cot x \, dx = -\csc x + c$	$\int \csc f(x) \cot f(x) dx = \frac{-\csc f(x)}{f'(x)} + c$
$\int \csc^2 x \, dx = -\cot x + c$	$\int \csc^2 f(x) dx = \frac{-\cot f(x)}{f'(x)} + c$
$\int \tan x \, dx = \ln\lvert\sec x\rvert + c$	$\int \tan x \, dx = \frac{\ln\lvert\sec f(x)\rvert}{f'(x)} + c$
$\int \sec x \, dx = \ln\lvert\sec x + \tan x\rvert + c$	$\int \sec x \; dx = \frac{\ln\lvert\sec f(x) + \tan f(x)\rvert}{f'(x)} + c$
$\int \cot x \, dx = \ln\lvert\sin x\rvert + c$	$\int \cot x \, dx = \frac{\ln\lvert\sin f(x)\rvert}{f'(x)} + c$
$\int \csc x \, dx = -\ln\lvert\csc x + \cot x\rvert + c$	$\int \csc x dx = \frac{-\ln\lvert\csc f(x) + \cot f(x)\rvert}{f'(x)} + c$

EXPONENTIAL FUNCTION

STANDARD FORM	GENERAL FORM Where : $f(x) = ax + b$
$\int e^x dx = e^x + c$	$\int e^{f(x)} dx = \frac{e^{f(x)}}{f'(x)} + c$

LOGARITHMIC FUNCTION

STANDARD FORM	GENERAL FORM Where : $f(x) = ax + b$
$\int \frac{1}{x} dx = \ln\lvert x\rvert + c$	$\int \frac{1}{f(x)} dx = \frac{\ln\lvert f(x)\rvert}{f'(x)} + c$

FOURIER TRANSFORM

Entry	$x(t)$	$X(f)$	$X(\omega)$
1	$\delta(t)$	1	1
2	$\mathrm{rect}(t)$	$\mathrm{sinc}(f)$	$\mathrm{sinc}\left(\frac{\omega}{2\pi}\right)$
3	$\mathrm{tri}(t)$	$\mathrm{sinc}^2(f)$	$\mathrm{sinc}^2\left(\frac{\omega}{2\pi}\right)$
4	$\mathrm{sinc}(t)$	$\mathrm{rect}(f)$	$\mathrm{rect}\left(\frac{\omega}{2\pi}\right)$
5	$\cos(2\pi\alpha t)$	$0.5[\delta(f+\alpha)+\delta(f-\alpha)]$	$\pi[\delta(\omega+2\pi\alpha)+\delta(\omega-2\pi\alpha)]$
6	$\sin(2\pi\alpha t)$	$j0.5[\delta(f+\alpha)-\delta(f-\alpha)]$	$j\pi[\delta(\omega+2\pi\alpha)-\delta(\omega-2\pi\alpha)]$
7	$e^{-\alpha t}u(t)$	$\frac{1}{\alpha+j2\pi f}$	$\frac{1}{\alpha+j\omega}$
8	$te^{-\alpha t}u(t)$	$\frac{1}{(\alpha+j2\pi f)^2}$	$\frac{1}{(\alpha+j\omega)^2}$
9	$e^{-\alpha\|t\|}$	$\frac{2\alpha}{\alpha^2+4\pi^2 f^2}$	$\frac{2\alpha}{\alpha^2+\omega^2}$
10	$e^{-\pi t^2}$	$e^{-\pi f^2}$	$e^{-\omega^2/4\pi}$
11	$\mathrm{sgn}(t)$	$\frac{1}{j\pi f}$	$\frac{2}{j\omega}$
12	$u(t)$	$0.5\delta(f)+\frac{1}{j2\pi f}$	$\pi\delta(\omega)+\frac{1}{j\omega}$
13	$e^{-\alpha t}\cos(2\pi\beta t)u(t)$	$\frac{\alpha+j2\pi f}{(\alpha+j2\pi f)^2+(2\pi\beta)^2}$	$\frac{\alpha+j\omega}{(\alpha+j\omega)^2+(2\pi\beta)^2}$
14	$e^{-\alpha t}\sin(2\pi\beta t)u(t)$	$\frac{2\pi\beta}{(\alpha+j2\pi f)^2+(2\pi\beta)^2}$	$\frac{2\pi\beta}{(\alpha+j\omega)^2+(2\pi\beta)^2}$
15	$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\frac{1}{T}\sum_{k=-\infty}^{\infty}\delta\left(f-\frac{k}{T}\right)$	$\frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta\left(\omega-\frac{2\pi k}{T}\right)$
16	$x_p(t)=\sum_{k=-\infty}^{\infty}X[k]e^{j2\pi k f_0 t}$	$\sum_{k=-\infty}^{\infty}X[k]\delta(f-kf_0)$	$\sum_{k=-\infty}^{\infty}2\pi X[k]\delta(\omega-k\omega_0)$

Property	$x(t)$	$X(f)$	$X(\omega)$
Similarity	$X(t)$	$x(-f)$	$2\pi x(-\omega)$
Time Scaling	$x(\alpha t)$	$\frac{1}{\|\alpha\|}X\left(\frac{f}{\alpha}\right)$	$\frac{1}{\|\alpha\|}X\left(\frac{\omega}{\alpha}\right)$
Folding	$x(-t)$	$X(-f)$	$X(-\omega)$
Time Shift	$x(t-\alpha)$	$e^{-j2\pi f\alpha}X(f)$	$e^{-j\omega\alpha}X(\omega)$
Frequency Shift	$e^{j2\pi\alpha t}x(t)$	$X(f-\alpha)$	$X(\omega-2\pi\alpha)$
Convolution	$x(t)\star h(t)$	$X(f)H(f)$	$X(\omega)H(\omega)$
Multiplication	$x(t)h(t)$	$X(f)\star H(f)$	$\frac{1}{2\pi}X(\omega)\star H(\omega)$
Modulation	$x(t)\cos(2\pi\alpha t)$	$0.5[X(f+\alpha)+X(f-\alpha)]$	$0.5[X(\omega+2\pi\alpha)+X(\omega-2\pi\alpha)]$
Derivative	$x'(t)$	$j2\pi fX(f)$	$j\omega X(\omega)$
Times-t	$-j2\pi tx(t)$	$X'(f)$	$2\pi X'(\omega)$
Integration	$\int_{-\infty}^{t} x(t)\,dt$	$\frac{1}{j2\pi f}X(f)+0.5X(0)\delta(f)$	$\frac{1}{j\omega}X(\omega)+\pi X(0)\delta(\omega)$
Conjugation	$x^*(t)$	$X^*(-f)$	$X^*(-\omega)$
Correlation	$x(t)\star\star y(t)$	$X(f)Y^*(f)$	$X(\omega)Y^*(\omega)$
Autocorrelation	$x(t)\star\star x(t)$	$X(f)X^*(f)=\|X(f)\|^2$	$X(\omega)X^*(\omega)=\|X(\omega)\|^2$

Fourier Transform Theorems	
Central ordinates	$x(0)=\int_{-\infty}^{\infty}X(f)\,df=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(\omega)\,d\omega \qquad X(0)=\int_{-\infty}^{\infty}x(t)\,dt$
Parseval's theorem	$E=\int_{-\infty}^{\infty}x^2(t)\,dt=\int_{-\infty}^{\infty}\|X(f)\|^2\,df=\frac{1}{2\pi}\int_{-\infty}^{\infty}\|X(\omega)\|^2\,d\omega$
Plancherel's theorem	$\int_{-\infty}^{\infty}x(t)y^*(t)\,dt=\int_{-\infty}^{\infty}X(f)Y^*(f)\,df=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(\omega)Y^*(\omega)\,d\omega$

Z-TRANSFORM TABLE

$x[n]$	$X(z)$	ROC
$\delta[n]$	1	All z
$u[n]$	$\frac{1}{1-z^{-1}}, \frac{z}{z-1}$	$\lvert z\rvert > 1$
$-u[-n-1]$	$\frac{1}{1-z^{-1}}, \frac{z}{z-1}$	$\lvert z\rvert < 1$
$\delta[n-m]$	z^{-m}	All z except 0 if $(m>0)$ or ∞ if $(m<0)$
$a^n u[n]$	$\frac{1}{1-az^{-1}}, \frac{z}{z-a}$	$\lvert z\rvert > \lvert a\rvert$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}, \frac{z}{z-a}$	$\lvert z < \lvert a\rvert$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}, \frac{az}{(z-a)^2}$	$\lvert z\rvert > \lvert a\rvert$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}, \frac{az}{(z-a)^2}$	$\lvert z\rvert < \lvert a\rvert$
$(n+1)a^n u[n]$	$\frac{1}{(1-az^{-1})^2}, \left[\frac{z}{z-a}\right]^2$	$\lvert z\rvert > \lvert a\rvert$
$(\cos \Omega_0 n)u[n]$	$\frac{z^2-(\cos \Omega_0)z}{z^2-(2\cos \Omega_0)z+1}$	$\lvert z\rvert > 1$
$(\sin \Omega_0 n)u[n]$	$\frac{(\sin \Omega_0)z}{z^2-(2\cos \Omega_0)z+1}$	$\lvert z\rvert > 1$
$(r^n \cos \Omega_0 n)u[n]$	$\frac{z^2-(r\cos \Omega_0)z}{z^2-(2r\cos \Omega_0)z+r^2}$	$\lvert z\rvert > r$
$(r^n \sin \Omega_0 n)u[n]$	$\frac{(r\sin \Omega_0)z}{z^2-(2r\cos \Omega_0)z+r^2}$	$\lvert z\rvert > r$
$\begin{cases} a^n & 0 \le n \le N-1 \\ 0 & \text{otherwise} \end{cases}$	$\frac{1-a^N z^{-N}}{1-az^{-1}}$	$\lvert z\rvert > 0$

COMPLEX ANALYSIS

$$\int_{\Gamma} f(z)dz = \int_a^b f(\Gamma(t))\Gamma'(t)dt$$

$$\oint_c \frac{f(z)}{z-z_0}dz = 2\pi i f(z_0)$$

Residue at pole z $= \lim_{z \to z_0} \left[(z-z_0)f(z)\right]$

Residue at pole z $= \frac{1}{(m-1)!} \lim_{z \to z_0} \left\{ \frac{d^{m-1}}{dz^{m-1}} \left[(z-z_0)^m f(z)\right] \right\}$

$$\oint_c f(z)dz = 2\pi j\left[\text{sum of residue of } f(z) \text{ at poles inside } C\right]$$

Common Sequences of the z-transform

a) $z\{u(n)\} = \frac{z}{z-1}$

b) $z\{n\} = \frac{z}{(z-1)^2}$

c) $z\{1\} = z\{u(n)\}$

d) $z\{a^n\} = \frac{z}{z-a}$

e) $z\{(n)^2\} = \frac{z^2+z}{(z-1)^3}$

Some Properties of the z-transform

Property	Sequence	Transform	ROC
	$x[n]$	$X(z)$	R
	$x_1[n]$	$X_1(z)$	R_1
	$x_2[n]$	$X_2(z)$	R_2
Linearity	$a_1x_1[n] + a_2x_2[n]$	$a_1X_1(z) + a_2X_2(z)$	$R' \supset R_1 \cap R_2$
Time shifting	$x[n-n_0]$	$z^{-n_0}X(z)$	$R' \supset R \cap \{0 < \lvert z\rvert < \infty\}$
Multiplication by z_0^n	$z_0^n x[n]$	$X\left(\frac{z}{z_0}\right)$	$R' = \lvert z_0\rvert R$
Multiplication by $e^{j\Omega_0 n}$	$e^{j\Omega_0 n}x[n]$	$X(e^{-j\Omega_0}z)$	$R' = R$
Time reversal	$x[-n]$	$X\left(\frac{1}{z}\right)$	$R' = \frac{1}{R}$
Multiplication by n	$nx[n]$	$-z\frac{dX(z)}{dz}$	$R' = R$
Accumulation	$\sum_{k=-\infty}^{n} x[n]$	$\frac{1}{1-z^{-1}}X(z)$	$R' \supset R \cap \{\lvert z\rvert > 1\}$
Convolution	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	$R' \supset R_1 \cap R_2$