CONFIDENTIAL

UNIVERSITI KUALA LUMPUR
Malaysian Institute of Marine Engineering Technology

FINAL EXAMINATION
JANUARY 2016 SESSION

SUBJECT CODE	:	LGB 12303
SUBJECT TITLE	:	MATHEMATICS 2
LEVEL	:	BACHELOR
TIME / DURATION	:	09.00 AM – 12.00 PM / 3 HOURS
DATE	:	30 MAY 2016

INSTRUCTIONS TO CANDIDATES

1. Please read the instructions given in the question paper CAREFULLY.
2. This question paper is printed on both sides of the paper.
3. Please write your answers on the answer booklet provided.
4. Answer should be written in blue or black ink except for sketching, graphic and illustration.
5. Answer FOUR (4) questions only
6. Answer all questions in English.

THERE ARE 6 PAGES OF QUESTIONS, INCLUDING THIS PAGE.

INSTRUCTION: Answer FOUR questions.

Please use the answer booklet provided.

Question 1

(a) The probability of a component failing in one year to excessive temperature is $\frac{1}{20}$ due to excessive vibration is $\frac{1}{25}$ and due to excessive humidity is $\frac{1}{50}$. Determine the probabilities that during a one-year period a component

i. fails due to excessive temperature and excessive vibration

(3 marks)

ii. fails due to excessive vibration or excessive humidity, and

(3 marks)

iii. will not fail because of both excessive temperature and excessive humidity

(3 marks)

(b) A box contains 74 brass washer, 86 steel washer and 40 aluminium washer. Three washers a drawn at random from the box without replacement. Determine the probability that all three are steel washers..

(6 marks)

(c) A batch of 40 components contains 5 units which are defective. If a component is drawn at random from the batch and tested and then a second component is drawn at random, calculate the probability of

i. having one defective component both with replacement, and

(5 marks)

ii. having one defective component both without replacement.

(5 marks)

Question 2

(a) Determine the ninth term and sixteenth term of the series $2, 7, 12, \ldots$.

(5 marks)

(b) Calculate four terms that can be inserted between 5 and 22.5 to form an arithmetic sequences.

(7 marks)

(c) The first term of a geometric sequences is 12 and the fifth term is 55. Determine the eighth term and eleventh term of the sequence.

(6 marks)

(d) Determine which term of the sequence $2187, 729, 243, \ldots$ is $\frac{1}{9}$.

(7 marks)

Question 3

(a) If $p = i + j - k$ and $q = i - 2j + 2k$, determine

i. $p \bullet q$ (3 marks)

ii. $p + q$ (2 marks)

iii. $|p + q|$, and (2 marks)

iv. $|p| + |q|$ (3 marks)

(b) Solve the equation $2j\left(j - \frac{d\beta}{dj}\right) = 5$, given $\beta = 2$ when $j = 1$.

(7 marks)

(c) Solve $\frac{1}{s}\frac{dr}{ds} = -4r + 2$

(8 marks)

Question 4

(a) Determine the general solution of $t\frac{dp}{dt} = 2 - 4t^3$

(5 marks)

(b) The equation of motion of a body oscillating at the end of a spring is

$$\frac{d^2x}{dt^2} + 100x = 0$$

where x is the displacement in meters of the body from its equilibrium position after time t seconds. Determine x in terms of t given that at time $t = 0, x = 2m$ and $\frac{dx}{dt} = 0$.

(10 marks)

(c) Solve the differential equation of $2\frac{d^2y}{dx^2} - 11\frac{dy}{dx} + 12y = 3x - 2$.

(10 marks)

Question 5

(a) By using the definition of Laplace transform, evaluate the Laplace transform of the function $f(t) = t$.

(5 marks)

(b) Determine $L^{-1}\left\{\dfrac{5s^2 + 8s - 1}{(s+3)(s^2+1)}\right\}$.

(10 marks)

(c) Use Laplace transform of solve the differential equation

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 13y = 0$$

given that when $x = 0$, $y = 3$ and $\dfrac{dy}{dx} = 7$.

(10 marks)

END OF QUESTIONS

DIFFERENTIATION

STANDARD FORM	GENERAL FORM
$\frac{d}{dx}(\sin x)=\cos x$	$\frac{d}{dx}(\sin f(x))=f'(x)\cos f(x)$
$\frac{d}{dx}(\cos x)=-\sin x$	$\frac{d}{dx}(\cos f(x))=-f'(x)\sin f(x)$
$\frac{d}{dx}(\tan x)=\sec^2 x$	$\frac{d}{dx}(\tan f(x))=f'(x)\sec^2 f(x)$
$\frac{d}{dx}(\csc x)=-\csc x\cot x$	$\frac{d}{dx}(\csc f(x))=-f'(x)\csc f(x)\cot f(x)$
$\frac{d}{dx}(\sec x)=\sec x\tan x$	$\frac{d}{dx}(\sec f(x))=f'(x)\sec f(x)\tan f(x)$
$\frac{d}{dx}(\cot x)=-\csc^2 x$	$\frac{d}{dx}(\cot f(x))=-f'(x)\csc^2 f(x)$

EXPONENTIAL FUNCTION

STANDARD FORM	GENERAL FORM
$\frac{d}{dx}e^x=e^x$	$\frac{d}{dx}e^{f(x)}=f'(x)e^{f(x)}$

LOGARITHMIC FUNCTION

STANDARD FORM	GENERAL FORM
$\frac{d}{dx}\ln x=\frac{1}{x}$	$\frac{d}{dx}\ln f(x)=\frac{f'(x)}{f(x)}$

INTEGRATION

STANDARD FORM	GENERAL FORM Where : $f(x)=ax+b$
$\int \cos x\,dx=\sin x+c$	$\int \cos f(x)dx=\frac{\sin f(x)}{f'(x)}+c$
$\int \sin x\,dx=-\cos x+c$	$\int \sin f(x)dx=\frac{-\cos f(x)}{f'(x)}+c$
$\int \sec^2 x\,dx=\tan x+c$	$\int \sec^2 f(x)dx=\frac{\tan f(x)}{f'(x)}+c$
$\int \sec x\tan x\,dx=\sec x+c$	$\int \sec f(x)\tan f(x)dx=\frac{\sec f(x)}{f'(x)}+c$
$\int \csc x\cot x\,dx=-\csc x+c$	$\int \csc f(x)\cot f(x)dx=\frac{-\csc f(x)}{f'(x)}+c$
$\int \csc^2 x\,dx=-\cot x+c$	$\int \csc^2 f(x)dx=\frac{-\cot f(x)}{f'(x)}+c$
$\int \tan x\,dx=\ln\lvert\sec x\rvert+c$	$\int \tan x\,dx=\frac{\ln\lvert\sec f(x)\rvert}{f'(x)}+c$
$\int \sec x\,dx=\ln\lvert\sec x+\tan x\rvert+c$	$\int \sec x\ dx=\frac{\ln\lvert\sec f(x)+\tan f(x)\rvert}{f'(x)}+c$
$\int \cot x\,dx=\ln\lvert\sin x\rvert+c$	$\int \cot x\,dx=\frac{\ln\lvert\sin f(x)\rvert}{f'(x)}+c$
$\int \csc x\,dx=-\ln\lvert\csc x+\cot x\rvert+c$	$\int \csc x dx=\frac{-\ln\lvert\csc f(x)+\cot f(x)\rvert}{f'(x)}+c$

EXPONENTIAL FUNCTION

STANDARD FORM	GENERAL FORM Where : $f(x)=ax+b$
$\int e^x dx=e^x+c$	$\int e^{f(x)}\,dx=\frac{e^{f(x)}}{f'(x)}+c$

LOGARITHMIC FUNCTION

STANDARD FORM	GENERAL FORM Where : $f(x)=ax+b$
$\int \frac{1}{x}dx=\ln\lvert x\rvert+c$	$\int \frac{1}{f(x)}dx=\frac{\ln\lvert f(x)\rvert}{f'(x)}+c$

FIRST AND SECOND ORDER DIFFERENTIAL EQUATION

If the roots of the auxiliary equation are:

(i) **real and different**, say $m = \alpha$ and $m = \beta$, then the general solution is

$$y = Ae^{\alpha x} + Be^{\beta x}$$

(ii) **real and equal, say** $m = \alpha$ twice, then the general solution is

$$y = (Ax + B)e^{\alpha x}$$

(iii) **complex**, say $m = \alpha \pm j\beta$, then the general solution is

$$y = e^{\alpha x}\{A \cos \beta x + B \sin \beta x\}$$

Table 51.1 Form of particular integral for different functions

Type	*Straightforward cases* Try as particular integral:	*'Snag' cases* Try as particular integral:
(a) $f(x)$ = a constant	$v=k$	$v=kx$ (used when C.F. contains a constant)
(b) $f(x)$ = polynomial (i.e. $f(x)=L+Mx+Nx^2+\cdots$ where any of the coefficients may be zero)	$v=a+bx+cx^2+\cdots$	
(c) $f(x)$ = an exponential function (i.e. $f(x)=Ae^{ax}$)	$v=ke^{ax}$	(i) $v=kxe^{ax}$ (used when e^{ax} appears in the C.F.) (ii) $v=kx^2e^{ax}$ (used when e^{ax} and xe^{ax} both appear in the C.F.)
(d) $f(x)$ = a sine or cosine function (i.e. $f(x)=a\sin px+b\cos px$, where a or b may be zero)	$v=A\sin px+B\cos px$	$v=x(A\sin px+B\cos px)$ (used when $\sin px$ and/or $\cos px$ appears in the C.F.)
(e) $f(x)$ = a sum e.g. (i) $f(x)=4x^2-3\sin 2x$ (ii) $f(x)=2-x+e^{3x}$	(i) $v=ax^2+bx+c+d\sin 2x+e\cos 2x$ (ii) $v=ax+b+ce^{3x}$	
(f) $f(x)$ = a product e.g. $f(x)=2e^x\cos 2x$	$v=e^x(A\sin 2x+B\cos 2x)$	

Table of Laplace Transforms

	$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$		$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$
1.	1	$\frac{1}{s}$	2.	e^{at}	$\frac{1}{s-a}$
3.	$t^n,\quad n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	4.	$t^p, p>-1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6.	$t^{n-\frac{1}{2}},\quad n=1,2,3,\ldots$	$\frac{1\cdot3\cdot5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7.	$\sin(at)$	$\frac{a}{s^2+a^2}$	8.	$\cos(at)$	$\frac{s}{s^2+a^2}$
9.	$t\sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	10.	$t\cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
11.	$\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$	12.	$\sin(at)+at\cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
13.	$\cos(at)-at\sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$	14.	$\cos(at)+at\sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$
15.	$\sin(at+b)$	$\frac{s\sin(b)+a\cos(b)}{s^2+a^2}$	16.	$\cos(at+b)$	$\frac{s\cos(b)-a\sin(b)}{s^2+a^2}$
17.	$\sinh(at)$	$\frac{a}{s^2-a^2}$	18.	$\cosh(at)$	$\frac{s}{s^2-a^2}$
19.	$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20.	$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21.	$e^{at}\sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22.	$e^{at}\cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23.	$t^n e^{at},\quad n=1,2,3,\ldots$	$\frac{n!}{(s-a)^{n+1}}$	24.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$
25.	$u_c(t)=u(t-c)$ Heaviside Function	$\frac{e^{-cs}}{s}$	26.	$\delta(t-c)$ Dirac Delta Function	e^{-cs}
27.	$u_c(t)f(t-c)$	$e^{-cs}F(s)$	28.	$u_c(t)g(t)$	$e^{-cs}\mathfrak{L}\{g(t+c)\}$
29.	$e^{ct}f(t)$	$F(s-c)$	30.	$t^n f(t),\quad n=1,2,3,\ldots$	$(-1)^n F^{(n)}(s)$
31.	$\frac{1}{t}f(t)$	$\int_s^\infty F(u)\,du$	32.	$\int_0^t f(v)\,dv$	$\frac{F(s)}{s}$
33.	$\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	34.	$f(t+T)=f(t)$	$\frac{\int_0^T e^{-st}f(t)\,dt}{1-e^{-sT}}$
35.	$f'(t)$	$sF(s)-f(0)$	36.	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
37.	$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\cdots-sf^{(n-2)}(0)-f^{(n-1)}(0)$			

Table Notes

1. This list is not a complete listing of Laplace transforms and only contains some of the more commonly used Laplace transforms and formulas.

2. Recall the definition of hyperbolic functions.

$$\cosh(t) = \frac{e^t + e^{-t}}{2} \qquad\qquad \sinh(t) = \frac{e^t - e^{-t}}{2}$$

3. Be careful when using normal trig function vs. hyperbolic functions. The only difference in the formulas is the q a^2 for the normal trig functions becomes a s a^2 for the hyperbolic functions!

4. Formula #4 uses the Gamma function which is defined as

$$\Gamma(t) = \int_0^\infty e^{-x} x^{t-1}\, dx$$

If n is a positive integer then,

$$\Gamma(n+1) = n!$$

The Gamma function is an extension of the normal factorial function. Here are a couple of quick facts for the Gamma function

$$\Gamma(p+1) = p\Gamma(p)$$

$$p(p+1)(p+2)\cdots(p+n-1) = \frac{\Gamma(p+n)}{\Gamma(p)}$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$