CONFIDENTIAL

UNIVERSITI KUALA LUMPUR
MALAYSIAN INSTITUTE OF MARINE ENGINEERING TECHNOLOGY

FINAL EXAMINATION
JANUARY 2016 SEMESTER

COURSE CODE	:	LEB30503
COURSE NAME	:	SIGNALS AND SYSTEMS
PROGRAMME NAME (FOR MPU: PROGRAMME LEVEL)	:	BACHELOR OF ENGINEERING TECHNOLOGY IN MARINE ELECTRICAL AND ELECTRONICS
DATE	:	
TIME	:	
DURATION	:	3 HOURS

INSTRUCTIONS TO CANDIDATES

NOTE: Instructions below to be edited to suit the needs of the intended course/examination.

1. Please CAREFULLY read the instructions given in the question paper.
2. This question paper has information printed on both sides of the paper.
3. This question paper consists of TWO (2) sections; Section A and Section B.
4. Answer ALL questions in Section A. For Section B, answer THREE (3) questions.
5. Please write your answers on the OMR answer script and answer booklet provided.
6. Answer all questions in English language ONLY.

THERE ARE 5 PAGES OF QUESTIONS, INCLUDING THIS PAGE.

SECTION A (Total: 40 marks)

INSTRUCTION: Answer ALL questions.
Please use the answer booklet provided.

Question 1

An LTI system generates the output,

$$y(t) = \left(e^{-2t} - e^{-3t}\right)u(t)$$

in response to the input $x(t) = e^{-2t}u(t)$

(a) Determine the unit impulse response *h(t)* of the system.

(10 marks)

(b) Sketch the amplitude $|H(\omega)|$ and the phase $\angle H(\omega)$ for the unit response from Question 1(a).

(10 marks)

You are required to use Fourier Transform method to answer this question.
(Course Learning Outcome: 2)

Question 2

(a) Consider the system as shown in Figure 2A:

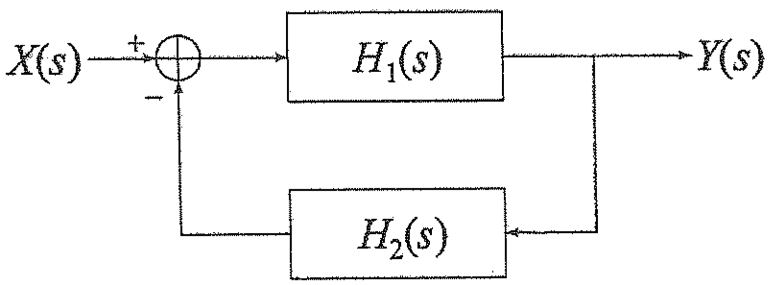

Figure 2A

where

$$H_1(s) = \frac{s}{(s+1)(s+a)} \quad \text{and} \quad H_2(s) = \frac{b}{s}$$

(i) Determine a and b such that the overall transfer function is

$$H(s)=\frac{s}{(s+4)(s+5)}$$

[5 marks]

(ii) Determine the output *y(t)* of the system with Question 2(a) transfer function to the unit-step input $x(t)=u(t)$.

[8 marks]

You are required to use Laplace Transform method to answer this question.
(Course Learning Outcome: 3)

(b) For an LTI system, we are given the z-transform of the input and output signals:

$$X(z)=\frac{1}{1+z^{-2}+z^{-4}}, \qquad Y(z)=\frac{1}{2}+\frac{1}{4}z^{-1}$$

Determine the impulse response h[n]. Sketch it.

(Course Learning Outcome: 3)

[7 marks]

SECTION B (Total: 60 marks)

INSTRUCTION: Answer THREE (3) questions ONLY.
Please use the answer booklet provided.

Question 3

(a) Consider a continuous-time system which has input of signals x(t) and output of $y(t) = x(t)u(t)$.

i. Is this system time invariant? Justify your answer.

[5 marks]

ii. Is this system linear? Justify your answer.

[5 marks]

(Course Learning Outcome: 1)

(b) Design an FIR filter for N = 9 whose frequency characteristic is given by

$$|H(e^{j\omega T})| = \begin{cases} 1 \;; & 0 \le \omega \le \pi/4T \\ 0 \;; & \pi/4T \le \omega \le 7\pi/4T \\ 1 \;; & 7\pi/4T \le \omega \le 2\pi/T \end{cases}$$

(Course Learning Outcome: 4)

(10 marks)

Question 4

(a) Consider the circuit shown in Figure 4A. Determine the input-output differential equation for this circuit in terms of input voltage *x(t)* and the output voltage *y(t)*.
(Course Learning Outcome: 1)

(10 marks)

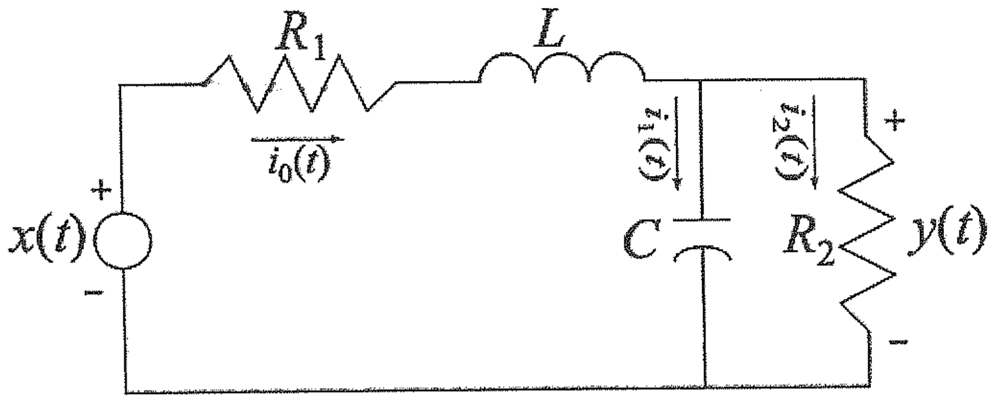

Figure 4A

(b) Design a low pass FIR filter with the following characteristics:

Passband 10kHz,

Stopband 11kHz, with attenuation of 50dB,

Sampling frequency 44kHz

Determine the causal impulse response h[n]. Use a Blackman window to design your filter.

(Course Learning Outcome: 4)

(10 marks)

Question 5

(a) Compute the convolution of the two signals as shown in Figure 5A using time-domain analysis method. Write down its analytical form and sketch its plot in the suitable axes. Show all your work.

(Course Learning Outcome: 1)

(12 marks)

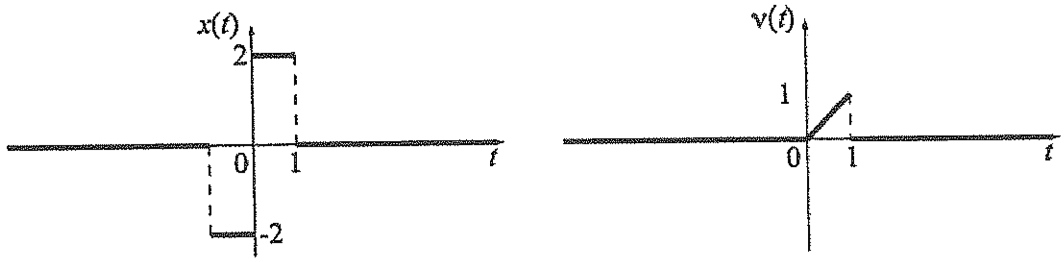

Figure 5A

(b) Determine total energy percentage in the frequency band $-4 < \omega < 4$ rad/s if 2Ω resistor has $i(t) = 2e^{-t}u(t)$ A.

(Course Learning Outcome: 4)

(8 marks)

Question 6

(a) Analyse the LTIC system that has an input response $x(t) = e^{-t}u(t)$ and unit impulse response $h(t) = 2e^{-t}u(t)$ and determine its zero-state response.

(Course Learning Outcome: 1)

(10 marks)

(b) Fauzi recorded the ocean wave using water level logger to observe wave wake generated by boat A at waterway behind MIMET. His professor help him to model the recorded signals into two cases as below:

i. Sinusoid 1: $x_1 = \sin(19.9\pi)$.

Sinusoid 2: $x_2 = 0.05\sin(40\pi)$.

ii. Sinusoid 1: $x_1 = \sin(19.9\pi)$.

Sinusoid 2: $x_2 = 0.05\sin(21\pi)$.

But he facing problem to interpret this model. Your task is to help him solve this problem by compare the measured spectra for that model which consisting of two sinusoidal components. Note that in both cases the record length is 10 seconds. Sketch also the spectra given in Question 6(b)i and Question 6(b)ii.

(Course Learning Outcome: 4)

(10

END OF EXAMINATION PAPER

Table of formulae for LEB30503 Signals and Systems

(For use during examination only)

Convolution Table

No.	$x_1(t)$	$x_2(t)$	$x_1(t) * x_2(t) = x_2(t) * x_1(t)$
1	$x(t)$	$\delta(t-T)$	$x(t-T)$
2	$e^{\lambda t}u(t)$	$u(t)$	$\frac{1-e^{\lambda t}}{-\lambda}u(t)$
3	$u(t)$	$u(t)$	$tu(t)$
4	$e^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(t)$	$\frac{e^{\lambda_1 t}-e^{\lambda_2 t}}{\lambda_1-\lambda_2}u(t) \quad \lambda_1 \neq \lambda_2$
5	$e^{\lambda t}u(t)$	$e^{\lambda t}u(t)$	$te^{\lambda t}u(t)$
6	$te^{\lambda t}u(t)$	$e^{\lambda t}u(t)$	$\frac{1}{2}t^2e^{\lambda t}u(t)$
7	$t^Nu(t)$	$e^{\lambda t}u(t)$	$\frac{N!\,e^{\lambda t}}{\lambda^{N+1}}u(t) - \sum_{k=0}^{N}\frac{N!\,t^{N-k}}{\lambda^{k+1}(N-k)!}u(t)$
8	$t^Mu(t)$	$t^Nu(t)$	$\frac{M!N!}{(M+N+1)!}t^{M+N+1}u(t)$
9	$te^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(t)$	$\frac{e^{\lambda_2 t}-e^{\lambda_1 t}+(\lambda_1-\lambda_2)te^{\lambda_1 t}}{(\lambda_1-\lambda_2)^2}u(t)$
10	$t^Me^{\lambda t}u(t)$	$t^Ne^{\lambda t}u(t)$	$\frac{M!\,N!}{(N+M+1)!}t^{M+N+1}e^{\lambda t}u(t)$
11	$t^Me^{\lambda_1 t}u(t)$ $\lambda_1 \neq \lambda_2$	$t^Ne^{\lambda_2 t}u(t)$	$\sum_{k=0}^{M}\frac{(-1)^kM!(N+k)!\,t^{M-k}e^{\lambda_1 t}}{k!(M-k)!(\lambda_1-\lambda_2)^{N+k+1}}u(t) + \sum_{k=0}^{N}\frac{(-1)^kN!(M+k)!\,t^{N-k}e^{\lambda_2 t}}{k!(N-k)!(\lambda_2-\lambda_1)^{M+k+1}}u(t)$
12	$e^{-\alpha t}\cos(\beta t+\theta)u(t)$	$e^{\lambda t}u(t)$	$\frac{\cos(\theta-\phi)e^{\lambda t}-e^{-\alpha t}\cos(\beta t+\theta-\phi)}{\sqrt{(\alpha+\lambda)^2+\beta^2}}u(t)$ $\phi=\tan^{-1}[-\beta/(\alpha+\lambda)]$
13	$e^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(-t)$	$\frac{e^{\lambda_1 t}u(t)+e^{\lambda_2 t}u(-t)}{\lambda_2-\lambda_1} \quad \mathrm{Re}\,\lambda_2 > \mathrm{Re}\,\lambda_1$
14	$e^{\lambda_1 t}u(-t)$	$e^{\lambda_2 t}u(-t)$	$\frac{e^{\lambda_1 t}-e^{\lambda_2 t}}{\lambda_2-\lambda_1}u(-t)$

Table of formulae for LEB30503 Signals and Systems

(For use during examination only)

Laplace Transform Table

No.	$x(t)$	$X(s)$
1	$\delta(t)$	1
2	$u(t)$	$\frac{1}{s}$
3	$tu(t)$	$\frac{1}{s^2}$
4	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5	$e^{\lambda t} u(t)$	$\frac{1}{s-\lambda}$
6	$te^{\lambda t} u(t)$	$\frac{1}{(s-\lambda)^2}$
7	$t^n e^{\lambda t} u(t)$	$\frac{n!}{(s-\lambda)^{n+1}}$
8a	$\cos bt\, u(t)$	$\frac{s}{s^2+b^2}$
8b	$\sin bt\, u(t)$	$\frac{b}{s^2+b^2}$
9a	$e^{-at} \cos bt\, u(t)$	$\frac{s+a}{(s+a)^2+b^2}$
9b	$e^{-at} \sin bt\, u(t)$	$\frac{b}{(s+a)^2+b^2}$
10a	$re^{-at} \cos(bt+\theta)\, u(t)$	$\frac{(r\cos\theta)s + (ar\cos\theta - br\sin\theta)}{s^2+2as+(a^2+b^2)}$
10b	$re^{-at} \cos(bt+\theta)\, u(t)$	$\frac{0.5re^{j\theta}}{s+a-jb} + \frac{0.5re^{-j\theta}}{s+a+jb}$
10c	$re^{-at} \cos(bt+\theta)\, u(t)$ $r = \sqrt{\frac{A^2c+B^2-2ABa}{c-a^2}}$ $\theta = \tan^{-1}\left(\frac{Aa-B}{A\sqrt{c-a^2}}\right)$ $b = \sqrt{c-a^2}$	$\frac{As+B}{s^2+2as+c}$
10d	$e^{-at}\left[A\cos bt + \frac{B-Aa}{b}\sin bt\right]u(t)$ $b = \sqrt{c-a^2}$	$\frac{As+B}{s^2+2as+c}$

Table of formulae for LEB30503 Signals and Systems

(For use during examination only)

Summary of Laplace Transform Operation

Operation	$x(t)$	$X(s)$
Addition	$x_1(t)+x_2(t)$	$X_1(s)+X_2(s)$
Scalar multiplication	$kx(t)$	$kX(s)$
Time differentiation	$\frac{dx}{dt}$	$sX(s)-x(0^-)$
	$\frac{d^2x}{dt^2}$	$s^2X(s)-sx(0^-)-\dot{x}(0^-)$
	$\frac{d^3x}{dt^3}$	$s^3X(s)-s^2x(0^-)-s\dot{x}(0^-)-\ddot{x}(0^-)$
	$\frac{d^nx}{dt^n}$	$s^nX(s)-\sum_{k=1}^{n}s^{n-k}x^{(k-1)}(0^-)$
Time integration	$\int_{0^-}^{t}x(\tau)\,d\tau$	$\frac{1}{s}X(s)$
	$\int_{-\infty}^{t}x(\tau)\,d\tau$	$\frac{1}{s}X(s)+\frac{1}{s}\int_{-\infty}^{0^-}x(t)\,dt$

Operation	$x(t)$	$X(s)$	
Time shifting	$x(t-t_0)u(t-t_0)$	$X(s)e^{-st_0}$	$t_0\geq 0$
Frequency shifting	$x(t)e^{s_0t}$	$X(s-s_0)$	
Frequency differentiation	$-tx(t)$	$\frac{dX(s)}{ds}$	
Frequency integration	$\frac{x(t)}{t}$	$\int_{s}^{\infty}X(z)\,dz$	
Scaling	$x(at),\,a\geq 0$	$\frac{1}{a}X\left(\frac{s}{a}\right)$	
Time convolution	$x_1(t)*x_2(t)$	$X_1(s)X_2(s)$	
Frequency convolution	$x_1(t)x_2(t)$	$\frac{1}{2\pi j}X_1(s)*X_2(s)$	
Initial value	$x(0^+)$	$\lim_{s\to\infty}sX(s)$	$(n>m)$
Final value	$x(\infty)$	$\lim_{s\to 0}sX(s)$	[poles of $sX(s)$ in LHP]

Table of formulae for LEB30503 Signals and Systems

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Fourier Transform Table

No.	$x(t)$	$X(\omega)$	
1	$e^{-at}u(t)$	$\dfrac{1}{a+j\omega}$	$a>0$
2	$e^{at}u(-t)$	$\dfrac{1}{a-j\omega}$	$a>0$
3	$e^{-a\|t\|}$	$\dfrac{2a}{a^2+\omega^2}$	$a>0$
4	$te^{-at}u(t)$	$\dfrac{1}{(a+j\omega)^2}$	$a>0$
5	$t^n e^{-at}u(t)$	$\dfrac{n!}{(a+j\omega)^{n+1}}$	$a>0$
6	$\delta(t)$	1	
7	1	$2\pi\delta(\omega)$	
8	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	
9	$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	
10	$\sin\omega_0 t$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$	
11	$u(t)$	$\pi\delta(\omega)+\dfrac{1}{j\omega}$	
12	$\operatorname{sgn} t$	$\dfrac{2}{j\omega}$	
13	$\cos\omega_0 t\, u(t)$	$\dfrac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\dfrac{j\omega}{\omega_0^2-\omega^2}$	
14	$\sin\omega_0 t\, u(t)$	$\dfrac{\pi}{2j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]+\dfrac{\omega_0}{\omega_0^2-\omega^2}$	
15	$e^{-at}\sin\omega_0 t\, u(t)$	$\dfrac{\omega_0}{(a+j\omega)^2+\omega_0^2}$	$a>0$
16	$e^{-at}\cos\omega_0 t\, u(t)$	$\dfrac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$a>0$

16	$e^{-at}\cos\omega_0 t\, u(t)$	$\dfrac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$a>0$
17	$\mathrm{rect}\left(\frac{t}{\tau}\right)$	$\tau\,\mathrm{sinc}\left(\frac{\omega\tau}{2}\right)$	
18	$\frac{W}{\pi}\,\mathrm{sinc}\,(Wt)$	$\mathrm{rect}\left(\frac{\omega}{2W}\right)$	
19	$\Delta\left(\frac{t}{\tau}\right)$	$\frac{\tau}{2}\,\mathrm{sinc}^2\left(\frac{\omega\tau}{4}\right)$	
20	$\frac{W}{2\pi}\,\mathrm{sinc}^2\left(\frac{Wt}{2}\right)$	$\Delta\left(\frac{\omega}{2W}\right)$	
21	$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\omega_0\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0)$	$\omega_0=\frac{2\pi}{T}$
22	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}\,e^{-\sigma^2\omega^2/2}$	

Table of formulae for LEB30503 Signals and Systems

(For use during examination only)

Summary of Fourier Transform Operation

Operation	$x(t)$	$X(\omega)$
Scalar multiplication	$kx(t)$	$kX(\omega)$
Addition	$x_1(t)+x_2(t)$	$X_1(\omega)+X_2(\omega)$
Conjugation	$x^*(t)$	$X^*(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Scaling (a real)	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{\omega}{a}\right)$
Time shifting	$x(t-t_0)$	$X(\omega)e^{-j\omega t_0}$
Frequency shifting (ω_0 real)	$x(t)e^{j\omega_0 t}$	$X(\omega-\omega_0)$

Operation	$x(t)$	$X(\omega)$
Time convolution	$x_1(t)*x_2(t)$	$X_1(\omega)X_2(\omega)$
Frequency convolution	$x_1(t)x_2(t)$	$\frac{1}{2\pi}X_1(\omega)*X_2(\omega)$
Time differentiation	$\frac{d^n x}{dt^n}$	$(j\omega)^n X(\omega)$
Time integration	$\int_{-\infty}^{t} x(u)\,du$	$\frac{X(\omega)}{j\omega}+\pi X(0)\delta(\omega)$

Table of formulae for LEB30503 Signals and Systems

(For use during examination only)

Z- Transform Table

	$f[k]$	$F[z]$
1	$\delta[k-j]$	z^{-j}
2	$u[k]$	$\dfrac{z}{z-1}$
3	$ku[k]$	$\dfrac{z}{(z-1)^2}$
4	$k^2u[k]$	$\dfrac{z(z+1)}{(z-1)^3}$
5	$k^3u[k]$	$\dfrac{z(z^2+4z+1)}{(z-1)^4}$
6	$\gamma^{k-1}u[k-1]$	$\dfrac{1}{z-\gamma}$
7	$\gamma^k u[k]$	$\dfrac{z}{z-\gamma}$
8	$k\gamma^k u[k]$	$\dfrac{\gamma z}{(z-\gamma)^2}$
9	$k^2\gamma^k u[k]$	$\dfrac{\gamma z(z+\gamma)}{(z-\gamma)^3}$
10	$\dfrac{k(k-1)(k-2)\cdots(k-m+1)}{\gamma^m m!}\gamma^k u[k]$	$\dfrac{z}{(z-\gamma)^{m+1}}$

$11a$	$\lvert\gamma\rvert^k \cos \beta k\, u[k]$		$\dfrac{z(z-\lvert\gamma\rvert\cos\beta)}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$
$11b$	$\lvert\gamma\rvert^k \sin \beta k\, u[k]$		$\dfrac{z\lvert\gamma\rvert\sin\beta}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$
$12a$	$r\lvert\gamma\rvert^k \cos(\beta k+\theta)u[k]$		$\dfrac{rz[z\cos\theta-\lvert\gamma\rvert\cos(\beta-\theta)]}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$
$12b$	$r\lvert\gamma\rvert^k \cos(\beta k+\theta)u[k]$	$\gamma=\lvert\gamma\rvert e^{j\beta}$	$\dfrac{(0.5re^{j\theta})z}{z-\gamma}+\dfrac{(0.5re^{-j\theta})z}{z-\gamma^*}$
$12c$	$r\lvert\gamma\rvert^k \cos(\beta k+\theta)u[k]$		$\dfrac{z(Az+B)}{z^2+2az+\lvert\gamma\rvert^2}$

$$r=\sqrt{\frac{A^2\lvert\gamma\rvert^2+B^2-2AaB}{\lvert\gamma\rvert^2-a^2}}$$

$$\beta=\cos^{-1}\frac{-a}{\lvert\gamma\rvert},\ \theta=\tan^{-1}\frac{Aa-B}{A\sqrt{\lvert\gamma\rvert^2-a^2}}$$

Table of formulae for LEB30503 Signals and Systems
(For use during examination only)
Summary of Z-Transform Operation

Operation	$f[k]$	$F[z]$
Addition	$f_1[k] + f_2[k]$	$F_1[z] + F_2[z]$
Scalar multiplication	$af[k]$	$aF[z]$
Right-shift	$f[k-m]u[k-m]$	$\frac{1}{z^m}F[z]$
	$f[k-m]u[k]$	$\frac{1}{z^m}F[z] + \frac{1}{z^m}\sum_{k=1}^{m} f[-k]z^k$
	$f[k-1]u[k]$	$\frac{1}{z}F[z] + f[-1]$
	$f[k-2]u[k]$	$\frac{1}{z^2}F[z] + \frac{1}{z}f[-1] + f[-2]$
	$f[k-3]u[k]$	$\frac{1}{z^3}F[z] + \frac{1}{z^2}f[-1] + \frac{1}{z}f[-2] + f[-3]$
Left-shift	$f[k+m]u[k]$	$z^m F[z] - z^m \sum_{k=0}^{m-1} f[k]z^{-k}$
	$f[k+1]u[k]$	$zF[z] - zf[0]$
	$f[k+2]u[k]$	$z^2F[z] - z^2f[0] - zf[1]$
	$f[k+3]u[k]$	$z^3F[z] - z^3f[0] - z^2f[1] - zf[2]$
Multiplication by γ^k	$\gamma^k f[k]u[k]$	$F\left[\frac{z}{\gamma}\right]$
Multiplication by k	$kf[k]u[k]$	$-z\frac{d}{dz}F[z]$
Time Convolution	$f_1[k] * f_2[k]$	$F_1[z]F_2[z]$
Frequency Convolution	$f_1[k]f_2[k]$	$\frac{1}{2\pi j}\oint F_1[u]F_2\left[\frac{z}{u}\right]u^{-1}\,du$
Initial value	$f[0]$	$\lim_{z\to\infty} F[z]$
Final value	$\lim_{N\to\infty} f[N]$	$\lim_{z\to 1}(z-1)F[z]$ poles of $(z-1)F[z]$ inside the unit circle.

Table of formulae for LEB30503 Signals and Systems
(For use during examination only)

Formulae

Butterworth Filters (BF):

Roots of the Butterworth polynomial,

$$s_m = -\sin[(2m-1)(\pi/2n)] + j\cos[(2m-1)(\pi/2n)] = \sigma_m + j\omega_m; \qquad m = 1,2,\ldots,2n$$

Chebyshev Filters (CF):

Minimum value of ripple (dB),

$$dB(\gamma) = 10\log_{10}(1+\varepsilon^2)$$

Roots of the Chebyshev polynomial,

$$s_m = -\left[\sin\left((2m-1)\left(\frac{\pi}{2n}\right)\right)\right]\sinh\left[\left(\frac{1}{n}\right)\sinh^{-1}\left(\frac{1}{\varepsilon}\right)\right] + j\left[\cos\left((2m-1)\left(\frac{\pi}{2n}\right)\right)\right]\cosh\left[\left(\frac{1}{n}\right)\sinh^{-1}\left(\frac{1}{\varepsilon}\right)\right]$$

Transfer function of CF for normalized frequency,

$$H_{cn}(s) = \frac{K}{(-1)^n \prod_{m=1}^{n}\left(\frac{s}{s_m} - 1\right)}$$, K = specified gain.

Transfer function of CF for denormalized frequency,

$$H_{cn}(s) = \frac{K}{(-1)^n \prod_{m=1}^{n}\left(\frac{s}{s_m\omega_c} - 1\right)}$$

FIR FILTER:

$$H_{lc}(z) = H_l(z)\ z^{-(N-1)/2}$$

$$H_l(z) = h(0) + \sum_{\ell=1}^{\frac{N-1}{2}} h(\ell T)\left(z^{\ell} + z^{-\ell}\right)$$