CONFIDENTIAL

UNIVERSITI KUALA LUMPUR

FINAL EXAMINATION

JANUARY 2016 SEMESTER

SUBJECT CODE	:	WQD10203
SUBJECT TITLE	:	TECHNICAL MATHEMATICS 2
LEVEL	:	DIPLOMA
TIME / DURATION	:	9.00 am – 11.30 am (2 ½ HOURS)
DATE	:	

INSTRUCTIONS TO CANDIDATES

1. Please read the instructions given in the question paper CAREFULLY.
2. This question paper is printed on both sides of the paper.
3. Please write your answers on the answer booklet provided.
4. Answer should be written in blue or black ink except for sketching, graphic and illustration.
5. This question paper consists of THREE (3) parts. Part A, B and C. Answer all questions in Part A and B. For Part C, answer two (2) questions only.
6. Answer all questions in English.
7. Formula Sheet is appended.

THERE ARE 9 PAGES OF QUESTIONS, EXCLUDING THIS PAGE.

PART A (Total: 15 marks)

MULTIPLE CHOICE QUESTIONS

INSTRUCTION: Answer ALL questions.

Please use the answer booklet provided.

1. Determine the amplitude of $y = -3\cos\left(2x - \frac{\pi}{4}\right)$.

   A. -3

   B. 3

   C. $2x$

   D. $-\frac{\pi}{4}$

2. Simplify the trigonometric expression $\frac{1}{\tan\theta} + \tan\theta$.

   A. $\frac{\cos\theta}{\sin\theta}$

   B. $\frac{1}{\sin\theta}$

   C. $\frac{1}{\cos\theta}$

   D. $\frac{1}{\sin\theta\cos\theta}$

3. Solve $3\sin\theta = 1$

   A. 19.47°, 160.53°

   B. 19.47°, 199.47°

   C. 160.47°, 199.47°

   D. 160.47°, 340.53°

4. Let $k(x)=\begin{cases} 4 & \text{for } x<-3 \\ x-2 & \text{for } -3\le x<2 \\ x^2 & \text{for } x\ge 2 \end{cases}$.

Determine $k(-5)$

A. 4

B. 1

C. -3

D. -1

5. Given $f(x)=3x$ and $g(x)=e^x$, compute $(g\circ f)(x)$.

A. $3xe^{3x}$

B. $3e^x$

C. $3xe^x$

D. e^{3x}

6. Determine $\lim_{r\to 3}\frac{r^2-9}{r-3}$

A. 0

B. 6

C. Undefined

D. 3

7. Differentiate y with respect to x for $y=e^{2x+1}$

A. $(2x+1)e^{2x+1}$

B. $2e^2$

C. $2e^{2x+1}$

D. $2xe^{2x+1}$

8. The differentiation of $\cot(3x^2-4)$ with respect to x is

A. $-6\csc^2(3x^2-4)$

B. $-4x\csc^2(3x^2-4)$

C. $-6x\csc^2(3x^2-4)$

D. $-4\csc^2(3x^2-4)$

9. Determine $\frac{dy}{dx}$ of $y = ln(2x-3)^4$.

A. $4ln(2x-3)^3$

B. $8ln(2x-3)^3$

C. $\frac{6}{(2x-3)^3}$

D. $\frac{8}{2x-3}$

10. Given $g(x) = \sqrt[3]{x^2-10}$. Determine $g'(x)$.

A. $\frac{2x}{3}(x^2-10)^{-\frac{2}{3}}$

B. $\frac{2x}{3}(x^2-10)^{\frac{2}{3}}$

C. $\frac{x}{2}(x^2-10)^{-\frac{2}{3}}$

D. $x(x^2-10)^{-\frac{2}{3}}$

11. Choose the correct formula of differentiation using quotient rule.

A. $\frac{g'(x)f(x)-g(x)f'(x)}{(g(x))^2}$

B. $f(x)g'(x)+g(x)f'(x)$

C. $\frac{g(x)f'(x)-g'(x)f(x)}{(g(x))^2}$

D. $f(x)g'(x)-g(x)f'(x)$

12. The integration of $y=(1-4x)^2$ is

A. $\frac{(1-4x)^3}{-12}+C$

B. $\frac{(1-4x)^3}{12}+C$

C. $\frac{(1-4x)}{8}+C$

D. $\frac{(1-4x)^3}{8}+C$

13. If $\int_a^b f(x)\,dx = d$ and $\int_b^c f(x)\,dx = -e$, evaluate $\int_a^c (f(x)+1)\,dx$.

A. $a + b + c + d - e$

B. $-a + b + c + d$

C. $a + b + c$

D. $-a + c + d - e$

14. $\int_1^2 (2x-3)\,dx$

A. -1

B. 0

C. 1

D. 2

15. If $\int_1^2 f(x)\,dx = 5$, then $\int_2^1 f(x)\,dx =$

A. 5

B. 1

C. 2

D. -5

PART B (Total: 45 marks)

INSTRUCTION: Answer ALL questions.
Please use the answer booklet provided.

Question 1

a) Show $\dfrac{1+\sec x}{\csc x} = \sin x + \tan x$.

[4 marks]

b) Given $\sin\alpha = -\dfrac{5}{13}$, determine the exact value of $\cos 2\alpha$ where α is in the third quadrant.

[3 marks]

Question 2

Given $f(x) = -2x+3$ and $g(x) = 2x^2+5x-1$, evaluate

a) $(f-g)(-1)$

[2 marks]

b) $(g \circ f)(x)$

[3 marks]

Question 3

Given the function defined by the Figure 1.

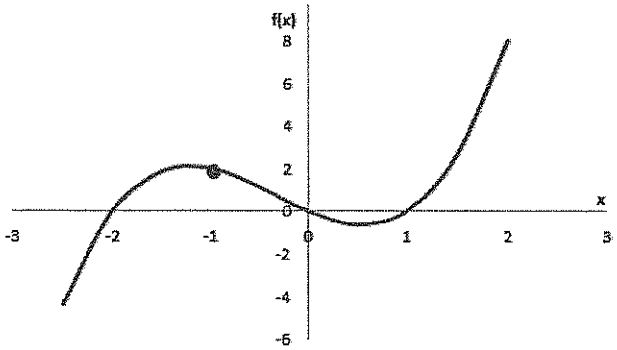

Figure 1

a) Determine each of the following;

i. $\lim_{x \to -1^-} f(x)$

[1 mark]

ii. $\lim_{x \to -1^+} f(x)$

[1 mark]

iii. $\lim_{x \to -1} f(x)$

[1 mark]

iv. $f(-1)$

[1 mark]

b) Determine whether the function $f(x)$ is continuous at $x = -1$.

[1 mark]

Question 4

Given $f(x) = \dfrac{x^3 + 3x}{x^2 - 2}$. Determine $f'(x)$.

[6 marks]

Question 5

Differentiate $f(x) = 2x^2 + 35$ by using the definition of derivative.

[Hint: use formula $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$]

[8 marks]

Question 6

Evaluate $\int_1^9 \frac{2x^2 + x^2\sqrt{x} - 1}{x^2}\, dx$.

[6 marks]

Question 7

Determine $\int \frac{x^2 + 1}{(x+3)(x-1)}\, dx$ by using integration by partial fraction.

[8 marks]

PART C (Total: 40 marks)

INSTRUCTION: Answer TWO questions.

Please use the answer booklet provided.

Question 1

a) Given $y = \frac{1}{2}\cos(3x)$

i. State the period and amplitude of y.

[2 marks]

ii. Sketch the graph for $0 \le x \le \pi$.

[8 marks]

b) Given $f(x) = \frac{3x+8}{2x-p}$, and $f(1) = 5$. Determine the value of p.

[5 marks]

c) Given $h(x) = \frac{x+4}{2x-5}$. Calculate the inverse function of $h(x)$ at the point $x = -1$.

[5 marks]

Question 2

a) If $y^5 - x^2y^3 = 1 + e^{x^2}$, determine $\frac{dy}{dx}$ at point $(0, 1)$.

[10 marks]

b) A water tank has a shape of an inverted circular cone with base radius 2 m and height 5m as shown in Figure 2. If water is being pumped into the tank at a rate of 0. 2 m³/min. Calculate the rate of change at which the water level is rising when the water is 3m deep.

[Hint Volume of cone $= \frac{1}{3}\pi r^2 h$]

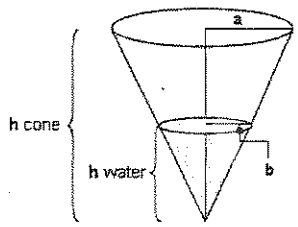

Figure 2

[10 marks]

Question 3

a) Evaluate $\int_0^1 x\sin(10x)\,dx$, by using integration by parts.

[7 marks]

b) A cup is made by rotating the area between $y = 2x^2$ and $y = x + 1$ with value of $x \geq 0$ around the x-axis in Figure 3. Calculate the volume of the material needed to make the cup.

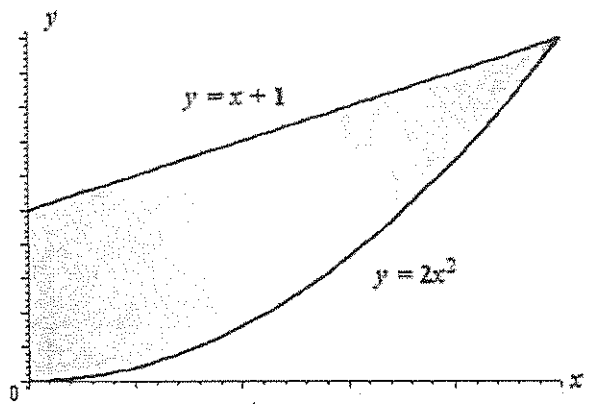

Figure 3

[13 marks]

END OF QUESTION

FORMULA SHEET

TRIGONOMETRY IDENTITIES

FUNDAMENTAL IDENTITIES	FORMULAS FOR NEGATIVES
$\csc\theta = \dfrac{1}{\sin\theta}$	$\sin(-\theta) = -\sin\theta$
$\sec\theta = \dfrac{1}{\cos\theta}$	$\cos(-\theta) = \cos\theta$
$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$	$\tan(-\theta) = -\tan\theta$
$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$	$\csc(-\theta) = -\csc\theta$
$\sin^2\theta + \cos^2\theta = 1$	$\sec(-\theta) = \sec\theta$
$1 + \tan^2\theta = \sec^2\theta$	$\cot(-\theta) = -\cot\theta$
$1 + \cot^2\theta = \csc^2\theta$	

ADDITION FORMULAS	SUBTRACTION FORMULAS
$\sin(A+B) = \sin A\cos B + \cos A\sin B$	$\sin(A-B) = \sin A\cos B - \cos A\sin B$
$\cos(A+B) = \cos A\cos B - \sin A\sin B$	$\cos(A-B) = \cos A\cos B + \sin A\sin B$
$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$	$\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

DOUBLE-ANGLE FORMULAS
$\sin 2\theta = 2\sin\theta\cos\theta$
$\cos 2\theta = \cos^2\theta - \sin^2\theta$ $\ldots\ldots\ldots = 1 - 2\sin^2\theta$ $\ldots\ldots\ldots = 2\cos^2\theta - 1$
$\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

DIFFERENTIATION

STANDARD FORM	GENERAL FORM
$\frac{d}{dx}(\sin x) = \cos x$	$\frac{d}{dx}(\sin f(x)) = f'(x)\cos f(x)$
$\frac{d}{dx}(\cos x) = -\sin x$	$\frac{d}{dx}(\cos f(x)) = -f'(x)\sin f(x)$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\frac{d}{dx}(\tan f(x)) = f'(x)\sec^2 f(x)$
$\frac{d}{dx}(\csc x) = -\csc x \cot x$	$\frac{d}{dx}(\csc f(x)) = -f'(x)\csc f(x)\cot f(x)$
$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\frac{d}{dx}(\sec f(x)) = f'(x)\sec f(x)\tan f(x)$
$\frac{d}{dx}(\cot x) = -\csc^2 x$	$\frac{d}{dx}(\cot f(x)) = -f'(x)\csc^2 f(x)$

EXPONENTIAL FUNCTION

STANDARD FORM	GENERAL FORM
$\frac{d}{dx}e^x = e^x$	$\frac{d}{dx}e^{f(x)} = f'(x)e^{f(x)}$

LOGARITHMIC FUNCTION

STANDARD FORM	GENERAL FORM
$\frac{d}{dx}\ln x = \frac{1}{x}$	$\frac{d}{dx}\ln f(x) = \frac{f'(x)}{f(x)}$

INTEGRATION

STANDARD FORM	GENERAL FORM Where : $f(x) = ax + b$
$\int \cos x\, dx = \sin x + c$	$\int \cos f(x) dx = \frac{\sin f(x)}{f'(x)} + c$
$\int \sin x\, dx = -\cos x + c$	$\int \sin f(x) dx = \frac{-\cos f(x)}{f'(x)} + c$
$\int \sec^2 x\, dx = \tan x + c$	$\int \sec^2 f(x) dx = \frac{\tan f(x)}{f'(x)} + c$
$\int \sec x \tan x\, dx = \sec x + c$	$\int \sec f(x) \tan f(x) dx = \frac{\sec f(x)}{f'(x)} + c$
$\int \csc x \cot x\, dx = -\csc x + c$	$\int \csc f(x) \cot f(x) dx = \frac{-\csc f(x)}{f'(x)} + c$
$\int \csc^2 x\, dx = -\cot x + c$	$\int \csc^2 f(x) dx = \frac{-\cot f(x)}{f'(x)} + c$

EXPONENTIAL FUNCTION

STANDARD FORM	GENERAL FORM Where : $f(x) = ax + b$
$\int e^x dx = e^x + c$	$\int e^{f(x)} dx = \frac{e^{f(x)}}{f'(x)} + c$

LOGARITHMIC FUNCTION

STANDARD FORM	GENERAL FORM Where : $f(x) = ax + b$
$\int \frac{1}{x} dx = \ln\lvert x\rvert + c$	$\int \frac{1}{f(x)} dx = \frac{\ln\lvert f(x)\rvert}{f'(x)} + c$

INTEGRATION BY PART

$\int u\, dv = uv - \int v\, du$