CONFIDENTIAL

UNIVERSITI KUALA LUMPUR
MALAYSIAN INSTITUTE OF INFORMATION TECHNOLOGY

FINAL EXAMINATION
JANUARY 2016 SEMESTER

COURSE CODE	:	IGD 21302
COURSE NAME	:	TECHNICAL MATHEMATICS 3
PROGRAMME LEVEL	:	DIPLOMA
DATE	:	23 MAY 2016
TIME	:	9.00 am – 11.30 am
DURATION	:	2 ½ HOURS

INSTRUCTIONS TO CANDIDATES

1. Please CAREFULLY read the instructions given in the question paper.
2. This question paper has information printed on both sides of the paper.
3. This question paper consists of TWO (2) sections; Section A and Section B. Answer ALL questions
4. Please write your answers on the answer booklet provided.
5. Answer all questions in English language ONLY.
6. Formula sheet has been appended for your reference.

THERE ARE 8 PAGES OF QUESTIONS, EXCLUDING THIS PAGE.

SECTION A (Total: 25 marks)

MULTIPLE CHOICE QUESTIONS

INSTRUCTION: Answer ALL questions.

Please use the answer booklet provided.

1. Which of the following statement referring to a **POPULATION**?

   A. Gross sales for 2008 of four fast – food chains

   B. Credit card debts of 90 families selected from a city.

   C. Total items produced on a machine for each year from 1999 to 2009.

   D. Salaries of CEOs of all companies in Kuala Lumpur.

2. Which of the following statement referring to a **continuous quantitative variable**?

   A. Minimum marks to get A.

   B. Income of 50 families lived in Kampung Buluh.

   C. Blood pressure readings.

   D. The number of cars using Jalan Sultan Ismail from 9.00 am – 10.00 am.

3. Following is the distribution of the weight of 70 mineral specimens collected on a field trip:

Weight (grams)	Number of specimens (f)
10 - 19	27
20 - 29	6
30 - 39	12
40 - 49	17
50 - 59	8

Calculate the **upper boundary** of the fourth class.

A. 40 C. 49.5

B. 44.5 D. 39.5

4. If a brown sack consists of 4 white balls and 3 black balls then probability of one randomly drawn ball will be white is

A. $\frac{1}{7}$ C. $\frac{3}{7}$

B. $\frac{4}{7}$ D. $\frac{4}{3}$

5. The monthly salary of two different companies are observed. There are 650 employees in Company A and the monthly salary is RM 2750. Meanwhile the number of employees in Company B are 700 and the monthly salary is RM 2500; then combined **arithmetic mean** is

A. RM 2320 C. RM 2620

B. RM 2625 D. RM 2420

6. A sample of adults were asked to report the numbers of hours they spent on the internet for the previous month. The results are listed below:

7, 12, 5, 33, 14, 8, 9, 12

What is the **mode** for this data set?

A. 33 C. 12

B. 7 D. 28

7. Determine the **median** of the data set mentioned in Question 6 above.

A. 23.5 C. 9

B. 14 D. 10.5

8. If $P(G) = 0.25$ and $P(T) = 0.63$, calculate the value of $P(G \cup T)$ if G and T are mutually exclusive events.

A. 0.88 C. 0.62

B. 0.75 D. 0.38

9. A and N are events defined on a sample space with $P(N) = 0.5$ and $P(A \mid N) = 0.15$. Calculate the probability of $P(A \cap N)$.

A. 0.75 C. 0.3

B. 0.075 D. 0.65

10. Define the nth term of a sequence given that the first term is 10 and the common difference is -3.

A. $a_n = 10 - 3n$
C. $a_n = 10 + 3n$
B. $a_n = 13 - 3n$
D. $a_n = 13 + 3n$

11. Determine the 5^{th} term of an arithmetic sequence in which the first term is -4 and the common difference is 4.

A. 12
C. 20
B. 8
D. 24

12. The first term and the last term of an arithmetic sequence are 4 and -68 respectively with the common difference is -3. How many term are there?

A. 23
C. 20
B. 22
D. 25

13. Calculate the sum of the first 25 terms in the following sequence, $4 + 8 + 12 + 16 + ...$

A. 196
C. 1300
B. 1250
D. 1350

14. What is the common ratio of the following sequence, 0.2, 0.04, 0.008, 0.0016, ...?

A. 0.04
C. 0.2
B. 5
D. 0.02

15. Which term of the geometric sequence 2, 6, 18, ... is 162?

A. 4
C. 6
B. 5
D. 3

16. A person has two parents, four grandparents, eight great grandparents, and so on. What is the total number of a person's ancestors in 5 generations?

A. 32
C. 30
B. 64
D. 62

17. Which of the following is the first order differential equation?

A. $x^2 + y^2 = 25$

B. $x^2 + yy' = 25$

C. $x^2 y + y'' = 25$

D. $x^2 y'' + y' = 25$

18. Given that $dy + 3ydx = e^{5x}dx$. By comparing to the standard first order differential equation, identify the function of $P(x)$ and $Q(x)$

A. $P(x) = 3, \quad Q(x) = e^{5x}$

B. $P(x) = 3y, \quad Q(x) = e^{5x}$

C. $P(x) = 3, \quad Q(x) = e^{5x}dx$

D. $P(x) = -3, \quad Q(x) = e^{5x}$

19. Determine the general solution of $y'' - 3y' - 10y = 0$.

A. $y = Ae^{-5x} + Be^{-2x}$

B. $y = e^{5x}(A + Bx)$

C. $y = Ae^{-5x} + Be^{-2x}$

D. $y = Ae^{5x} + Be^{-2x}$

20. The following are second order linear differential equation except for:

A. $\frac{d^2y}{dx^2} - 10\frac{dy}{dx} = -25y$

B. $y'' - 6y' + 3y = 0$

C. $y''' - 5y'' + y' = 0$

D. $y'' + 9y = 10y'$

21. By using separation of variables, the general solution of the differential equation $\frac{dy}{dx} = xy$ is

A. $y = \frac{x^2}{2} + C$

B. $\ln|y| = \frac{x^2}{2} + C$

C. $y = x + C$

D. $\ln|y| = x + C$

22. It is shown that $y = e^{2x}$ is a solution of the differential equation $y'' - 5y' + 6y = 0$. What is the nature of this solution?

A. General solution
B. Particular solution
C. Many solution
D. Identical solution

23. Determine the inverse Laplace transform of $F(s) = \frac{s}{s^2 + 4}$.

A. $f(x) = 1 - e^{-2x}$
B. $f(x) = \cos 2x$
C. $f(x) = \sin 2x$
D. $f(x) = xe^{-2x}$

24. The Laplace transform of $y = x\sin 3x$ is

A. $\frac{2s}{(s^2 + 9)^2}$
B. $\frac{3s}{(s^2 + 9)^2}$
C. $\frac{6s}{(s^2 + 9)^2}$
D. $\frac{6}{(s^2 + 9)^2}$

25. The Laplace transform of $y = 1 - \cos x$ is

A. $\frac{1}{s(s^2 + 1)}$
B. $\frac{1}{s(s^2 + 1)^2}$
C. $\frac{1}{(s^2 + 1)}$
D. $\frac{1}{s} - \frac{1}{s^2 + 1}$

SECTION B (Total: 75 marks)

INSTRUCTION: Answer ALL questions.

Please use the answer booklet provided.

Question 1

The following frequency distribution table shows the central processing unit (CPU) time in seconds required to run a program using a statistical package:

Table 1 : Estimated time (in seconds)

Estimate CPU time (in seconds)	Frequency
0 – 2.0	2
2.1 – 4.0	5
4.1 – 6.0	14
6.1 – 8.0	8
8.1 – 10.0	1

By showing all the working required, answer each of the following:

a) Calculate the **mean**.

[4 marks]

b) Determine the **variance** and the **standard deviation** of the data.

[8 marks]

c) Calculate the **mode** of the data.

[6 marks]

Question 2

Prince Hotel has done a survey to determine whether its service is satisfactory to its customer. The result is summarized in the table below. A guest is randomly selected from these 100 people.

Table 2 : Guest satisfactory in Prince Hotel

Category	Satisfied	Unsatisfied
Female	42	2
Male	40	16

Determine:

a) the probability that this guest is satisfied.

[3 marks]

b) the probability that this guest is a male given that he is also satisfied with the service.

[3 marks]

c) the probability that this guest is a unsatisfied given that the guest is a male. Draw a **tree diagram** to illustrate the conditional probability

[7 marks]

Question 3

a) Zul is arranging 870 chairs for opening ceremony in the hall tonight. He start with 15 chairs in the first row, 18 in the second row, and 21 in the third row and so on.

i. Determine the number of rows chairs arranged in this hall.

[8 marks]

ii. If the hall can accommodate 45 rows of chairs, calculate the seating capacity in this hall.

[5 marks]

b) The first term and the fourth term of a geometric sequence are 5 and 135 respectively. Determine the common ratio of the sequence.

[5 marks]

Question 4

a) Show that $y = 5x + x^2$ is a solution of the differential equation $xy' - y = x^2$.

[3 marks]

b) By using integrating factor, solve the differential equation $\frac{dy}{dx} - 5y = e^{6x}$.

[8 marks]

Question 5

If $y(0) = 0$, by using the Laplace transform, solve the differential equation $y' + 2y = 4e^{-x} + 2$.

[15 marks]

END OF EXAM QUESTION

FORMULA

STATISTICS & PROBABILITY

1. **Mean for Ungrouped** data (sample):

$$\bar{x} = \frac{\sum_{1}^{n} x_i}{n}$$

2. **Mean for grouped** data (sample):

$$\bar{x} = \frac{\sum f_i x_i}{\sum_{i=1}^{n} f_i}$$

3. **Median for Grouped** data:

$$\tilde{x} = L + \left[\frac{\frac{\left(\sum_{1}^{n} f_i\right)+1}{2} - f_L}{f} \right] \times C$$

Where:

L = lower boundary of median class

f_L = cumulative frequency before the median class

c = class width

f = frequency of median class

4. **Mode for Grouped** data:

$$\hat{x} = m + \left(\frac{d_1}{d_1 + d_2} \right) \times C$$

Where:

m = lower boundary of mode class

d_1 = the difference between the frequency of mode class and the frequency of the class before mode class

d_2 = the difference between the frequency of mode class and the frequency of the class after mode class

c = class width

5. **Variance for Ungrouped** data (sample):

$$s^2 = \frac{\sum_{1}^{n} (x_i - \bar{x})^2}{n-1} \quad \textbf{OR}$$

$$s^2 = \frac{\sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n}}{n-1}$$

6. **Variance for Grouped** data (sample)

$$s^2 = \frac{1}{\sum_{1}^{n} f_i - 1} \left(\sum_{1}^{n} f_i x_i^2 - \frac{\left(\sum_{1}^{n} f_i x_i\right)^2}{\sum_{1}^{n} f_i} \right)$$

[Standard deviation is the square root of the variance]

7. Probability Laws

i. If A and B are mutually exclusive events then,

$$P(A \cup B) = P(A) + P(B);\ P(A \cap B) = 0$$

ii. If A and B are **NOT** mutually exclusive events then,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

iii. Conditional probability for A given B has occurred is:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)};\quad \text{if } P(B) > 0$$

SEQUENCE & SERIES

1. Arithmetic Sequence		
i. The nth term of: $a_n = a_1 + (n-1)d$	ii. The sum of the first n term $S_n = \frac{n}{2}(2a_1 + (n-1)d)$	iii. The sum of the finite terms: $S_n = \frac{n}{2}(a_1 + a_n)$
2. Geometric Sequence		
i. The nth term of: $a_n = a_1 r^{n-1}$	ii. The sum of the first n term: $S_n = a_1\left(\frac{1-r^n}{1-r}\right),\ r<1$ $S_n = a_1\left(\frac{r^n-1}{r-1}\right),\ r>1$	iii. The sum to infinity: $S_\infty = \frac{a_1}{1-r}$, if $-1<r<1$

DIFFERENTIAL EQUATION

1. A **first order linear differential equation** is an equation of the form;

$$\frac{dy}{dx} + Py = Q$$

where $P(x)$ and $Q(x)$ are continuous functions.

2. **Integrating factor** :

$$ye^{\int P dx} = \int Qe^{\int P dx} dx + C$$

3. **Second Order Linear Differential Equation**

Nature of roots	Condition	General Solution
i. Real and unequal roots, m_1, m_2	$b^2 - 4ac > 0$	$y = Ae^{m_1 x} + Be^{m_2 x}$
ii. Real and equal roots, m	$b^2 - 4ac = 0$	$y = e^{mx}(A + Bx)$
iii. Complex roots, $m_1 = \alpha + j\omega$ $m_2 = \alpha - j\omega$	$b^2 - 4ac < 0$	$y = e^{\alpha x}(A\cos\omega x + B\sin\omega x)$

4. **Table of derivatives**		5. **Table of integrals**	
Function	Derivatives	Function $f(x)$	Indefinite integral $\int f(x)dx$
Constant	0	Constant, k	$kx + c$
x	1	x	$\frac{x^2}{2} + c$
kx	k	kx^2	$\frac{kx^3}{3} + c$
x^n	nx^{n-1}	x^n	$\frac{x^{n+1}}{n+1} + c \; ; \; n \neq 1$
kx^n	knx^{n-1}	kx^n	$\frac{kx^{n+1}}{n+1} + c \; ; \; n \neq 1$
e^x	e^x	e^x	$e^x + c$
$e^{g(x)}$	$e^{g(x)}.g'(x)$	$e^{g(x)}$	$\frac{e^{g(x)}}{g'(x)} + c$
$\ln x$	$\frac{1}{x}$	$x^{-1} = \frac{1}{x}$	$\ln\lvert x\rvert + c$
$\ln g(x)$	$\frac{1}{x}.g'(x)$	$\frac{1}{ax+b}$	$\frac{1}{a}\ln\lvert ax+b\rvert + c$
$f(x).g(x)$	$f(x).g'(x) + g(x).f'(x)$	$\sin x$	$-\cos x + c$
$\frac{f(x)}{g(x)}$	$\frac{g(x).f'(x) - f(x).g'(x)}{[g(x)]^2}$	$\cos x$	$\sin x + c$
$\frac{1}{g(x)}$	$\frac{-g'(x)}{[g(x)]^2}$	$\tan x$	$\ln\lvert\sec x\rvert + c$
$\sin x$	$\cos x$	$\cot x$	$\ln\lvert\sin x\rvert + c$
$\cos x$	$-\sin x$	$\sec^2 x$	$\tan x + c$

LAPLACE TRANSFORM

1. Laplace transform of the **first derivative** function $L(f') = sL(f) - f(0)$ **OR** $L(y') = sL(y) - y(0)$	2. Laplace transform of the **second derivative** function $L(f'') = s^2L(f) - sf(0) - f'(0)$ **OR** $L(y'') = s^2L(y) - sy(0) - y'(0)$

3. Table of Laplace Transform

	Function, $f(x)$	Laplace transform, $L\{f(x)\} = F(s)$		Function, $f(x)$	Laplace transform, $L\{f(x)\} = F(s)$
1.	a	$\frac{a}{s}$	12.	$1 - \cos ax$	$\frac{a^2}{s(s^2 + a^2)}$
2.	x	$\frac{1}{s^2}$	13.	$ax - \sin ax$	$\frac{a^3}{s^2(s^2 + a^2)}$
3.	x^2	$\frac{2}{s^3}$	14.	$e^{-ax} - e^{-bx}$	$\frac{b-a}{(s+a)(s+b)}$
4.	e^{ax}	$\frac{1}{s-a}$	15.	$ae^{-ax} - be^{-bx}$	$\frac{s(a-b)}{(s+a)(s+b)}$
5.	e^{-ax}	$\frac{1}{s+a}$	16.	$[(b-a)x + 1]e^{-ax}$	$\frac{s+b}{(s+a)^2}$
6.	xe^{-ax}	$\frac{1}{(s+a)^2}$	17.	$\sin ax - ax \cos ax$	$\frac{2a^3}{(s^2 + a^2)^2}$
7.	$1 - e^{-ax}$	$\frac{a}{s(s+a)}$	18.	$x \sin ax$	$\frac{2as}{(s^2 + a^2)^2}$
8.	$e^{-ax}(1 - ax)$	$\frac{s}{(s+a)^2}$	19.	$x \cos ax$	$\frac{s^2 - a^2}{(s^2 + a^2)^2}$
9.	$x^n e^{-ax}$	$\frac{n!}{(s+a)^{n+1}}$	20.	$\sin ax + ax \cos ax$	$\frac{2as^2}{(s^2 + a^2)^2}$
10.	$\cos ax$	$\frac{s}{s^2 + a^2}$	21.	$e^{-ax} \sin bx$	$\frac{b}{(s+a)^2 - b^2}$
11.	$\sin ax$	$\frac{a}{s^2 + a^2}$	22.	$e^{-ax} \cos bx$	$\frac{s+a}{(s+a)^2 - b^2}$